MATHEMATICS PAPER 2

KCSE 2011

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SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1 Use logarithms, correct to 4 decimal places, to evaluate

$$\sqrt[3]{\frac{83.46 \times 0.0054}{1.56^2}}$$

(4 marks)

2 Three grades A, B, and C of rice were mixed in the ratio 3:4:5. The cost per kg of each of the grades A, B and C were Ksh 120, Ksh 90 and Ksh 60 respectively.
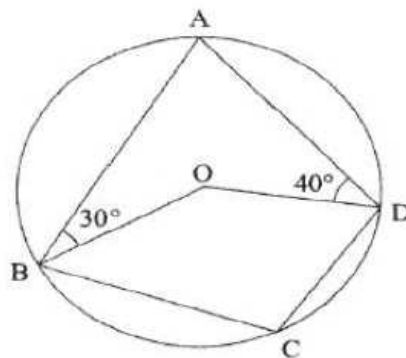Calculate:

(a) the cost of one kg of the mixture; (2 marks)

(b) the selling price of 5 kg of the mixture given that the mixture was sold at 8% profit. (2 marks)

3 Make s the subject of the formula.

$$w = \sqrt[3]{\frac{s+t}{s}}$$

(3 marks)

4 (a) Solve the inequalities $2x - 5 > -11$ and $3 + 2x \leq 13$, giving the answer as a combined inequality. (3 marks)

(b) List the integral values of x that satisfy the combined inequality in (a) above. (1 mark)

5 In the figure below, ABCD is a cyclic quadrilateral. Point O is the centre of the circle. Angle ABO = 30° and angle ADO = 40°.


Calculate the size of angle BCD. (2 marks)

6 The ages in years of five boys are 7, 8, 9, 10 and 11 while those of five girls are 4, 5, 6, 7 and 8. A boy and a girl are picked at random and the sum of their ages is recorded.

(a) Draw a probability space to show all the possible outcomes. (1 mark)

(b) Find the probability that the sum of their ages is at least 17 years. (1 mark)

7 The vertices of a triangle are A(1,2), B(3,5) and C(4,1). The coordinates of C' the image of C under a translation vector T, are (6,–2).

(a) Determine the translation vector T. (1 mark

(b) Find the coordinates of A' and B' under translation vector T. (2 marks

8 Write sin 45° in the form $\frac{1}{\sqrt{a}}$ where a is a positive integer. Hence simplify $\frac{\sqrt{8}}{1+\sin 45°}$, leaving the answer in surd form. (3 marks

9 The radius of a spherical ball is measured as 7 cm, correct to the nearest centimetre. Determine, to 2 decimal places, the percentage error in calculating the surface area of the ball. (4 marks

10 (a) In the figure below, lines NA and NB represent tangents to a circle at points A and B. Use a pair of compasses and ruler only to construct the circle. (2 marks
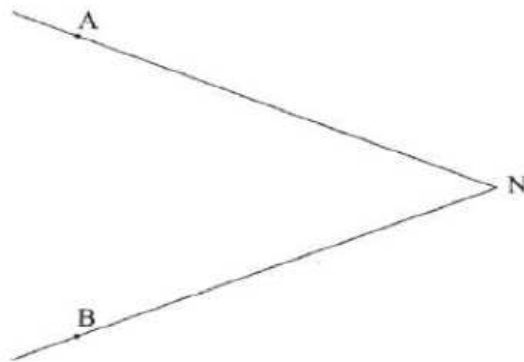
(b) Measure the radius of the circle. (1 mark

11 Expand and simplify the expression.

$$\left(a+\tfrac{1}{2}\right)^4 + \left(a-\tfrac{1}{2}\right)^4$$

(3 marks)

12   The figure below represents a scale drawing of a rectangular piece of land, RSTU.
      RS = 9 cm and ST = 7 cm.
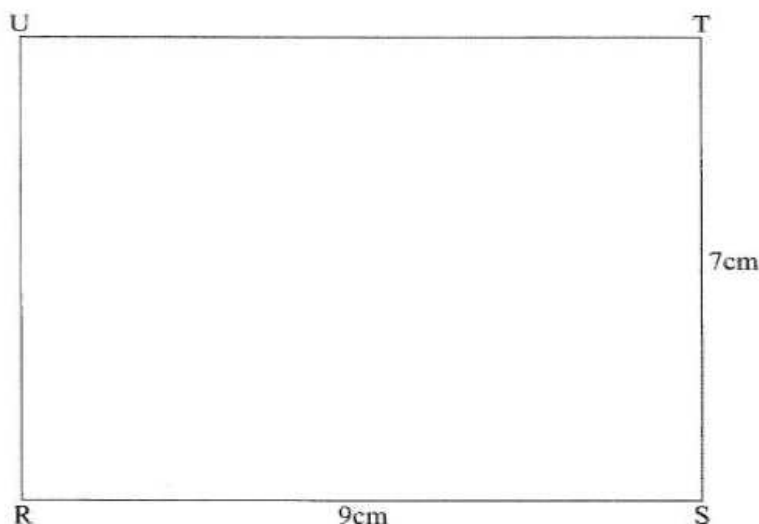
An electric post P, is to be erected inside the piece of land. On the scale drawing, shade the
possible region in which P would lie such that PU > PT and PS ≤ 7 cm.   (3 marks)

13   Vector **OP**= 6**i** + **j** and **OQ** = –2**i** + 5**j**. A point N divides **PQ** internally in the ratio 3:1.
      Find **PN** in terms of **i** and **j**.   (3 marks)

14   A point M (60°N, 18°E) is on the surface of the earth. Another point N is situated at a
      distance of 630 nautical miles east of M.
      Find:
      (a)   the longitude difference between M and N;   (2 marks)

      (b)   the position of N.   (1 mark)

15   The equation of a circle centre (a,b) is $x^2 + y^2 - 6x - 10y + 30 = 0$.
      Find the values of a and b.   (3 marks)

16   The table below shows values of x and y for the function y = 2 sin 3x° in the range
      0° ≤ x ≤ 150°.

$x°$	0	15	30	45	60	75	90	105	120	135	150
y	0	1.4	2	1.4	0	-1.4	-2	-1.4	0	1.4	2

      (a)   On the grid provided, draw the graph of y = 2 sin 3x.   (2 marks)

(b) From the graph determine the period. (1 mark)

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17 The cash price of a laptop was Ksh 60 000. On hire purchase terms, a deposit of Ksh 7 500 was paid followed by 11 monthly installments of Ksh 6 000 each.

    (a) Calculate:
        (i) the cost of a laptop on hire purchase terms; (2 marks)

        (ii) the percentage increase of hire purchase price compared to the cash price. (2 marks)

    (b) An institution was offered a 5% discount when purchasing 25 such laptops on cash terms. Calculate the amount of money paid by the institution. (2 marks)

    (c) Two other institutions, X and Y, bought 25 such laptops each. Institutions X bought the laptops on hire purchase terms. Institution Y bought the laptops on cash terms with no discount by securing a loan from a bank. The bank charged 12% p.a. compound interest for two years.
Calculate how much more money institution Y paid than institution X. (4 marks)

18 The first, fifth and seventh terms of an Arithmetic Progression (AP) correspond to the first three consecutive terms of a decreasing Geometric Progression (G.P). The first term of each progression is 64, the common difference of the AP is d and the common ratio of the G.P is r.

    (a) (i) Write two equations involving d and r. (2 marks)

        (ii) Find the values of d and r. (4 marks)

    (b) Find the sum of the first 10 terms of:

        (i) the Arithmetic Progression (A.P); (2 marks)

        (ii) the Geometric Progression (G.P). (2 marks)

19   The vertices of a rectangle are A(−1,−1), B(−4,−1), C(−4,−3) and D(−1,−3).

(a)   On the grid provided, draw the rectangle and its image A' B' C' D' under a
transformation whose matrix is $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.   (4 marks)
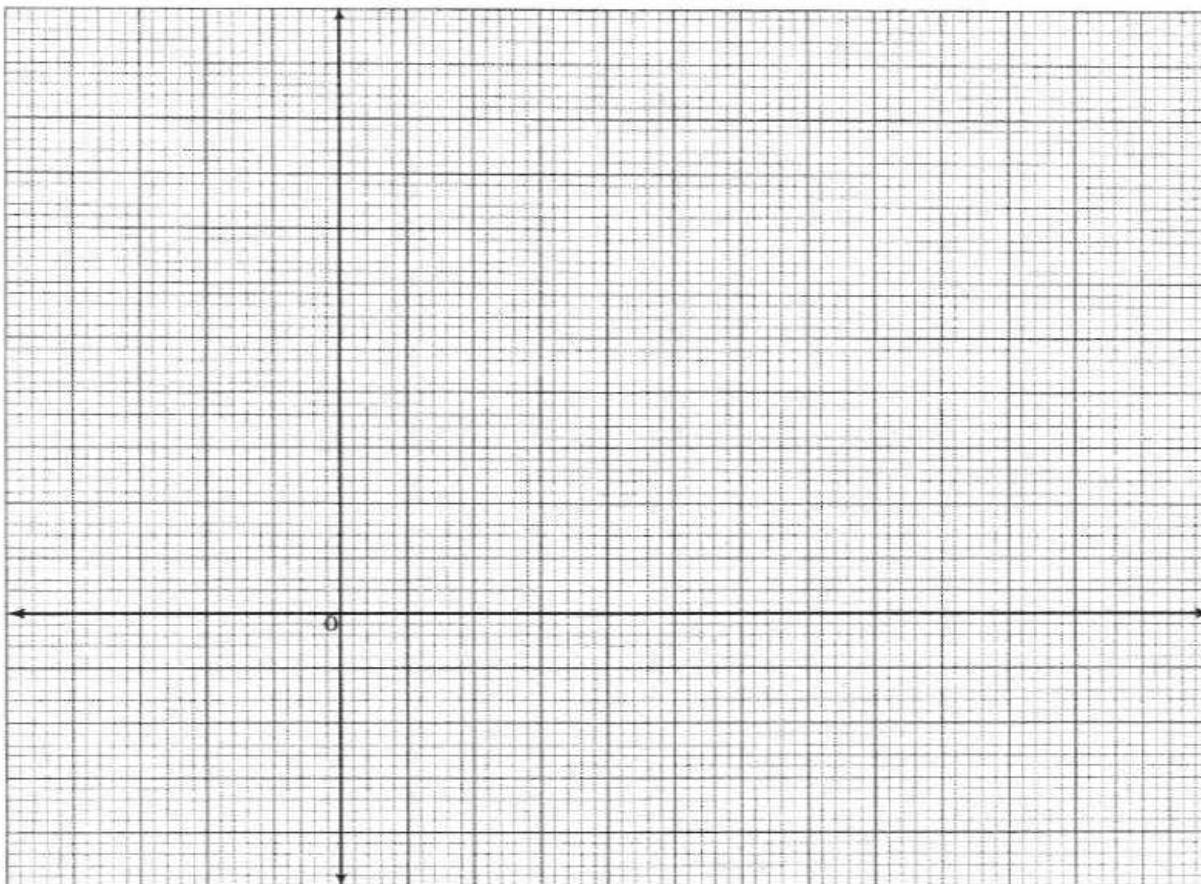
(b)   A" B" C" D" is the image of A' B' C' D' under a transformation matrix,

$$P = \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{pmatrix}.$$

(i)   Determine the coordinates of A", B", C" and D".   (2 marks)

(ii)   On the same grid draw the quadrilateral A" B" C" D".   (1 mark)

(c)   Find the area of A" B" C" D".   (3 marks)

20   A parent has two children whose age difference is 5 years. Twice the sum of the ages of the two children is equal to the age of the parent.

   (a)   Taking x to be the age of the elder child, write an expression for:

   (i)   the age of the younger child; (1 mark)

   (ii)   the age of the parent. (1 mark)

   (b)   In twenty years time, the product of the children's ages will be 15 times the age of their parent.

   (i)   Form an equation in x and hence determine the present possible ages of the elder child. (4 marks)

   (ii)   Find the present possible ages of the parent. (2 marks)

   (iii)   Determine the possible ages of the younger child in 20 years time. (2 marks)

21   The table below shows values of x and some values of y for the curve $y = x^3 + 2x^2 - 3x - 4$ for $-3 \leq x \leq 2$.

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-4.0	-0.4		1.6	0		-4.0	-4.9			6

   (a)   Complete the table by filling in the missing values of y, correct to 1 decimal place. (2 marks)

   (b)   On the grid provided, draw the graph of $y = x^3 + 2x^2 - 3x - 4$.
   Use the scale: 1 cm represents 0.5 units on x-axis.
   1 cm represents 1 unit on y-axis. (3 marks)

(c)   Use the graph to:

   (i)   solve the equation $x^3 + 2x^2 - 3x - 4 = 0$;   (3 marks)

      (ii)   estimate the coordinates of the turning points of the curve.   (2 marks)

22   The figure below represents a rectangular based pyramid VABCD. AB = 12 cm and AD = 16 cm. Point O is vertically below V and VA = 26 cm.
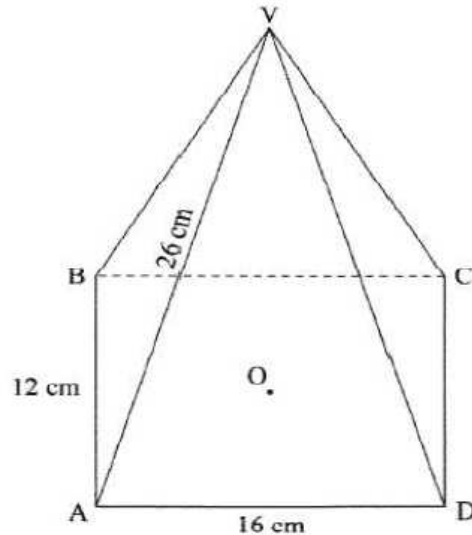
Calculate:
(a)   the height, VO, of the pyramid;   (4 marks)

(b)   the angle between the edge VA and the plane ABCD;   (3 marks)

(c)   the angle between the planes VAB and ABCD.   (3 marks)

23   The cost C, of producing n items varies partly as n and partly as the inverse of n. To produce two items it costs Ksh 135 and to produce three items it costs Ksh 140.
Find:

   (a)   the constants of proportionality and hence write the equation connecting C and n;   (5 marks)

   (b)   the cost of producing 10 items;   (2 marks)

(c)   the number of items produced at a cost of Ksh 756.   (3 marks)

24   A building contractor has two lorries, P and Q, used to transport at least 42 tonnes of sand to a building site. Lorry P carries 4 tonnes of sand per trip while lorry Q carries 6 tonnes of sand per trip. Lorry P uses 2 litres of fuel per trip while lorry Q uses 4 litres of fuel per trip. The two lorries are to use less than 32 litres of fuel. The number of trips made by lorry P should be less than 3 times the number of trips made by lorry Q. Lorry P should make more than 4 trips.

    (a)   Taking x to represent the number of trips made by lorry P and y to represent the number of trips made by lorry Q, write the inequalities that represent the above information.

                (4 marks)

    (b)   On the grid provided, draw the inequalities and shade the unwanted regions.

                (4 marks)

    (c)   Use the graph drawn in (b) above to determine the number of trips made by lorry P and by lorry Q to deliver the greatest amount of sand.   (2 marks)

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1   Given that $m = \dfrac{3}{0.089^2}$ and $n = \dfrac{1}{\sqrt{82.49}}$,

use a calculator to find:

(a)   the value of m and the value of n;   (2 marks)

(b)   the value of $m + n$ to 4 significant figures.   (1 mark)

2   Given that $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$, find $3\mathbf{a} - 5\mathbf{b}$.   (3 marks)

3   The mass of an object is 0.36 kg and its density is 2.5g/cm³.
Calculate the volume of the object in cm³.   (2 marks)

4   Make T the subject of the formula,   (3 marks)

$$P = \sqrt{\dfrac{S(T-R)}{A}}$$

5   A trader mixes two types of fruit juices A and B in the ratio 2:5. Type A costs Ksh 140 per litre and type B costs Ksh 105 per litre.
Find the selling price of the mixture per litre if the trader makes a 20% profit.   (4 marks)

6 The table below shows the ages of a group of students.

Age in years	14	15	16	17	18
Number of students	2	6	14	16	10

Draw a pie chart to represent the above information. (3 marks)

7 Given that $P = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $R = P^2Q$, determine R. (3 marks)

8 Find the number which must be added to the quadratic expression $x^2 + 6x + 1$ to make it a perfect square. (3 marks)

9 A point P is located 10 cm from the centre of a circle of radius 6 cm. Calculate the length of a tangent drawn from P to the circle. (2 marks)

10 A bag contains balls of identical size of which 36 are blue and the rest yellow. When a a ball is drawn at random from the bag, the probability that it is yellow is $\frac{2}{5}$. Calculate the number of yellow balls in the bag. (3 marks)

11 In a triangular plot of land ABC, BC = 18 m, AC = 10 m and angle ACB = 80°. Calculate to 2 decimal places:

(a) the length AB; (2 marks)

(b) the size of angle CAB. (2 marks)

12 Below is part of an income tax table for monthly income in a certain year.

Monthly Taxable income in Ksh	Tax Rate in each shilling
Up to Ksh 10 164	10%
From Ksh 10 165 up to Ksh 19 740	15%
From Ksh 19 741 up to Ksh 29 316	20%

In that year Wambita's monthly taxable salary was Ksh 18 000. He was entitled to a monthly personal relief of Ksh 1162.
Calculate the monthly income tax paid. (4 marks)

13 Two towns on the equator differ in local time by 6 hours. Find the distance in km, between the two towns.
(Take the circumference of the earth to be 40 000 km) (3 marks)

14 The first term of an arithmetic progression (A.P) is 7 and the 17th term is 81. There are 15 other terms between them.
Calculate:

(a) the sum of the 17 terms; (2 mark:

(b) the sum of the 15 middle terms of the A.P. (2 marks

15 The matrix $\begin{pmatrix} 4 & 1 \\ 6 & 2 \end{pmatrix}$ maps point P onto its image P′.
Determine the matrix that maps the image P′ onto P. (3 marks

16 Corresponding value of x and y in a given relation are as shown in the table below.

x	15	18	23	30	35	40	45	53
y	0.10	0.18	0.23	0.34	0.40	0.50	0.55	0.74

On the grid provided, plot all the points and draw the line of best fit. (3 marks
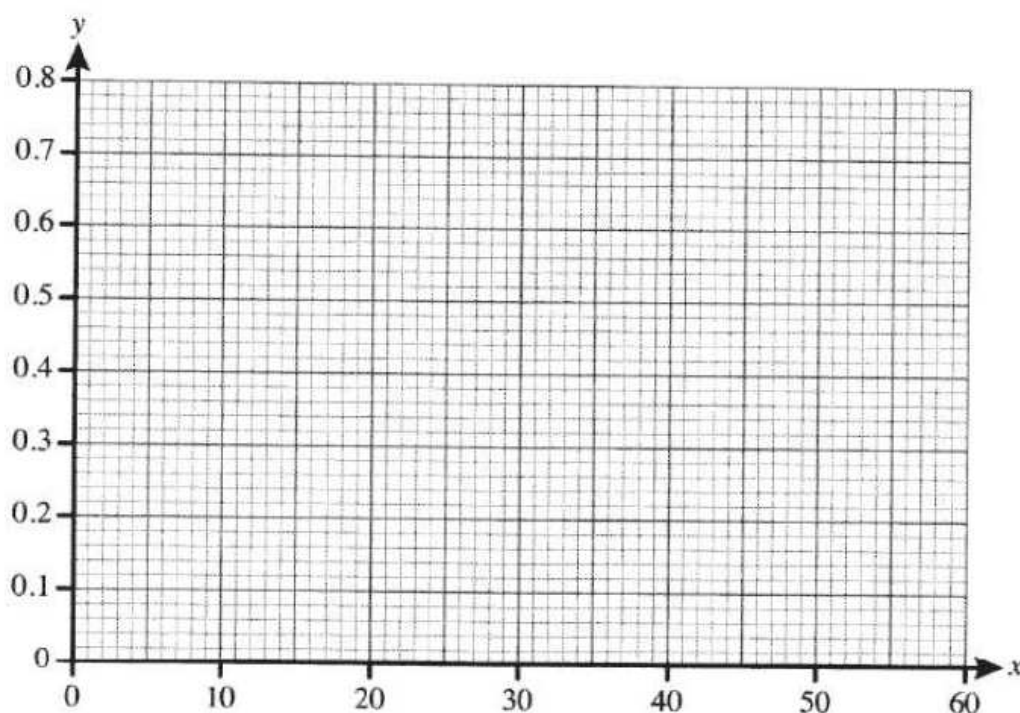
Answer only five questions in this section in the spaces provided.

17   Three machines P, Q and R take 8 hours, 12 hours and 16 hours respectively to complete a task. The three machines were set to work together for $1\frac{1}{2}$ hours. Machine Q was then switched off while machines P and R were left to complete the remaining task.

(a)   Find the fraction of the task done by P, Q and R in the first hour.   (2 marks)

(b)   Calculate the fraction of the task:

   (i)   done by P, Q and R in $1\frac{1}{2}$ hours;   (2 marks)

   (ii)   left after $1\frac{1}{2}$ hours.   (2 marks)

(c)   Determine the time, in hours and minutes, taken by machines P and R to complete the remaining task.   (4 marks)

18   The third and the sixth terms of a geometric progression are 18 and 486 respectively. Calculate:

(a)   the common ratio;   (3 marks)

(b)   the first term;   (2 marks)

(c)   the sum of the ninth and tenth terms;   (3 marks)

19   The coordinates of points A, B and C are A(2,2), B(5,6) and C(9,8). Point D is such that **AD** = 3 **BC**.

(a)   Find:

   (i)   **BC**;   (2 marks)

   (ii)   the coordinates of point D.   (4 marks)

(b)   Given that T is the midpoint of **AD**, find:

   (i)   the coordinates of point T;   (2 marks)

   (ii)   the magnitude of **TC**, correct to 2 significant figures.   (2 marks)

20   Two towns, T and U are 36 km apart. A cyclist travelled from town T to town U at an average speed of x km/h. On his journey back from town U to town T his average speed was $(x + 3)$ km/h.

(a)   Write down an expression in terms of x for the time in hours the cyclist took to travel from:

   (i)   town T to town U;   (1 mark)

   (ii)   town U to town T.   (1 mark)

(b)   The journey from town T to town U took one hour longer than the journey from town U to town T. Form an equation in *x* and hence determine the average speed of the cyclist on his journey back from town U to town T.   (5 marks)

(c)   Calculate to one decimal place, the cyclists' average speed for the whole journey from town T to town U and back.   (3 marks)

21   Matata, a horticulture farmer, carried out the following transactions in the month of April 2010.

April   1: Had Ksh 8 000 carried forward from March 2010.
        4: Bought 2 bags of fertilizer @Ksh 1 750.
        5: Paid Ksh 600 for water.
        9: Bought spraying chemicals for Ksh 1 500.
      12: Received Ksh 15 000 from the sale of bananas.
      15: Sold cabbages for Ksh 5 000.
      16: Paid wages to two casual workers at Ksh 1 500 each.
      20: Sold tomatoes for Ksh 9 500.
      24: Paid Ksh 840 for electricity.
      25: Bought seeds for Ksh 450.
      28: Sold onions for Ksh 2 500.
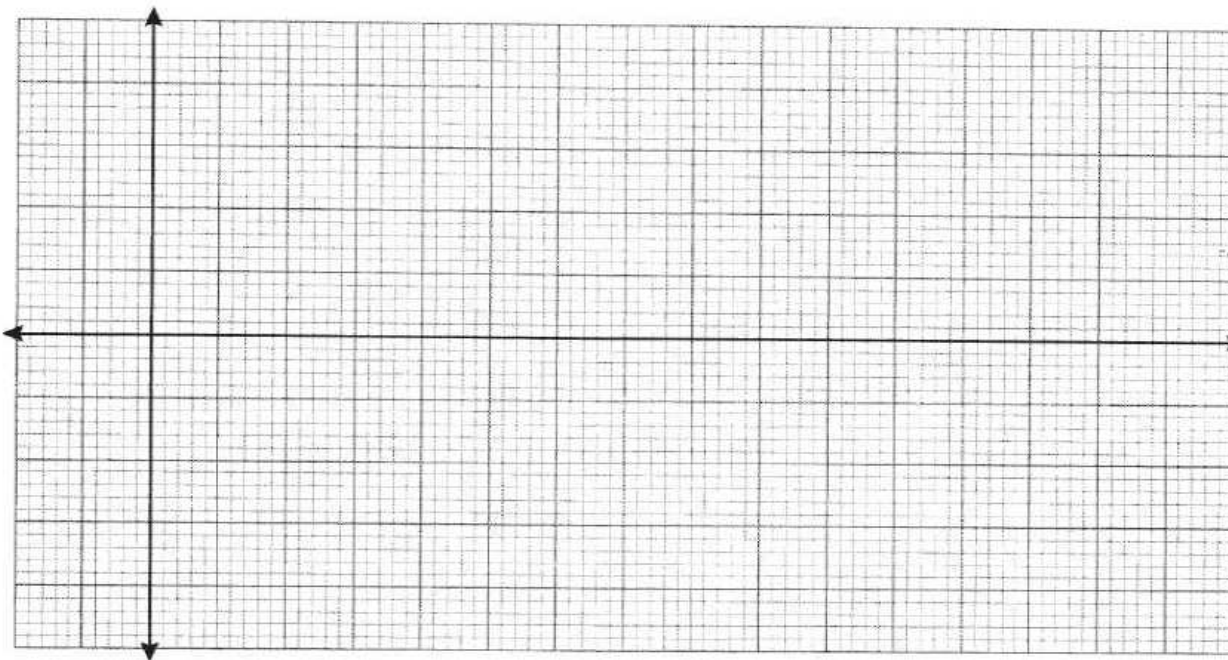   30: Bought a spray pump for Ksh 7 500.

Prepare a single column cash book for Matata's transactions and balance it as at 30th April, 2010.   (10 marks)

22 (a) (i) Complete the table below for $y = 2\sin x°$. (2 marks)

$x°$	0	30	60	90	120	150	180	210	240	270	300	330	360
$y = 2\sin x°$	0	1			1.73		0	−1				−1	0

(ii) On the grid below draw the graph of $y = 2\sin x°$ for $0° \le x \le 360°$. Use
1 cm for 30° on the x-axis and 2 cm for 1 unit on the y-axis. (4 marks)

(b) Use the graph to find:

(i) the values of x for which $y = 1.5$; (2 marks)

(ii) the range of values of x for which $2\sin x° > 1$. (2 marks)