Marking Scheme KCSE 2016

MATHEMATICS

PAPER 1

P1 Q1.

Without using a calculator evaluate, $\frac{-2(5+3)-9\div3+5}{-3\times-5+-2\times4}$ (3 mks)

Answer

$$\frac{-2(5+3-9+3+5}{-3\times^{-}5+(^{-}2)\times4}=\frac{-14}{7}=-2$$

P1 Q 2

Simplify

$$\frac{p^2+2pq+q^2}{p^3-pq^2+p^2q-q^3}$$

Answer

$$\frac{(p+q)(p+q)}{p(p^2-q^2)+q(p^2-q^2)}$$

$$=\frac{(p+q)(p+q)}{(p+q)(p+q)(p-q)}$$

$$=\frac{1}{p+q}$$

P1 Q 3

The external length, width and height of an open rectangular container are 41 cm 21 cm and 15.5cm respectively. The thickness of the material making the container is 5mm. If the container has 8 litres of water, calculate the internal height above the water level. (4 mks)

Answer

Internal Dimensions: 40, 20 and 15

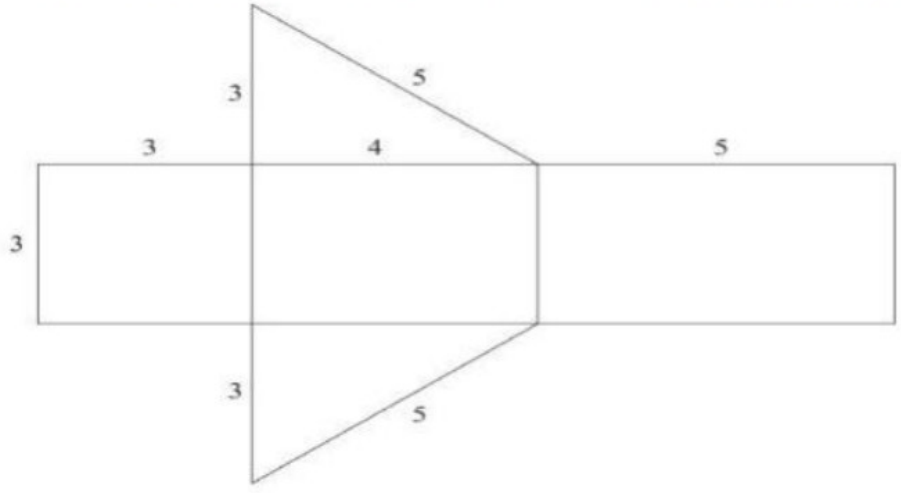
Volume unoccupied = $40\times20\times15-8000$

$= 4000$

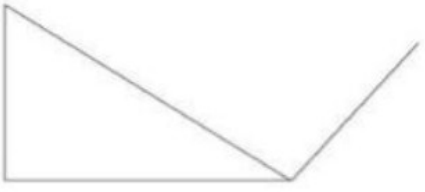
Height above water level = $\frac{4000}{40\times20}=5\text{cm}$

P1 Q4

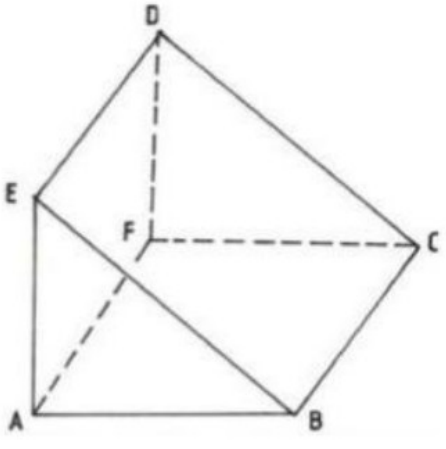
The figure below shows a net of a solid. (measurements are in centimetres).



Below is a part of the sketch of the solid whose net is shown above. Complete the sketch of the solid, showing the hidden edges with broken lines.
(3 marks



Answer



PP1 Q 5

Given that **OA** = 2**i** + 3**j** and **OB** = 3**i** – 2**j**. Find the magnitude of **AB** to one decimal place (3 marks)

Answer

$$\begin{pmatrix} -3 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -5 \end{pmatrix}$$

$$\text{magnitude} = \sqrt{1^2 + (-5)^2}$$

$$= \sqrt{26} \simeq 5.1$$

P1 Q6

A bus travelling at an average speed of 63km/h left a station at 8.15 a.m. A car later left the same station at 9.00 a.m. and caught up with the bus at 10.45 a.m. Find the average speed of the car.

Answer

Distance covered by bus

$= 63 \times (10.45 - 8.15)$

$= 63 \times 2.5$

$= 157.5$

Speed of car

$$= \frac{157.5}{1.75}$$

$= 90$ km/h

P1 Q7

Given that x is an acute angle and cos x = $\frac{2\sqrt{5}}{5}$, find without using mathematical tables or a calculator, tan (90 – x)^0. (2 marks)

Answer

$l^2 = 5^2 - (2\sqrt{5})^2 = 5$

$l = \sqrt{5}$

$\therefore \tan(90-x)^\circ = \frac{2\sqrt{5}}{\sqrt{5}}$ *or* 2

P1 Q 8

Without using mathematical tables or a calculator,evaluate

$27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{-\frac{1}{4}}$.

Answer

$$27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{-\frac{1}{4}} = \left(3^3\right)^{\frac{2}{3}} \times \left(\frac{3^4}{2^4}\right)^{-\frac{1}{4}}$$

$$= 3^2 \times \left(\frac{3}{2}\right)^{-1}$$

$$= 3^2 \times \frac{2}{3}$$

$$= 6$$

P1 Q 9

A minor arc of a circle subtends an angle of 105° at the centre of the circle. If the radius of the circle is 8.4 cm, find the length of the major arc. (Take $\pi = \frac{22}{7}$)

(3 marks)

Answer

Angle for major arc $= 360 - 105$

$= 255°$

Length of arc $= \frac{255}{360} \times 2 \times 8.4 \times \frac{22}{7}$

$= 37.4$ cm

P1 Q10

The gradient of the tangent to the curve $y = ax^3 + bx$ at the point (1,1) is -5. Calculate the values of a and b (4 marks)

Answer

$\frac{dy}{dx} = 3ax^2 + b$

$3a + b = -5$

$a + b = 1$

$a = -3$

$b = 4$

P1 Q11

A line with gradient of -3 passes through the points (3, k) and (k,8). Find the value of k and hence express the equation of the line in the form of ax + by = c, where a, b, and c are constants.

Answer

$\frac{k-8}{3-k} = \frac{-3}{1}$

$k-8 = -9 + 3k$

$2k = 1$

$K = \frac{1}{2}$

$\frac{y-8}{x-\frac{1}{2}} = \frac{-3}{1}$

$y-8 = -3(x-\frac{1}{2})$

$y-8 = -3x + \frac{3}{2}$

$2y-16 = 6x + 3$

$6x + 2y = 19$

P1 Q 12

Points L and M are equidistant from another point K. The bearing of L from K is 330^0. The bearing of M from K is 220^0. Calculate the bearing of M from L (3 marks)

Answer

$\angle LKM = 110°$

$\angle KLM = 35°$ OR $KML = 35°$

Bearing is 185°

P1 Q 13

In this question, mathematical tables should not be used.

A Kenyan bank buys and sells foreign currencies as shown below

Buying	***Selling***
(In Kenya shillings)	***In Kenya Shillings***
1 Hong Kong dollar	
9.74	***9.77***
1 South African rand	
12.03	***12.11***

A tourist arrived in Kenya with 105 000 Hong Kong dollars and changed the whole amount to Kenyan shillings. While in Kenya, she spent Kshs 403 897 and changed the balance to South African rand before leaving for South Africa. Calculate the amount, in South African rand that she received. (3 marks)

Answer

105000×9.74

$= sh\ 1022700$

$\frac{1022700 - 403879}{12.11}$

$= \frac{618821}{12.11}$

$= 51100$ rands

P1 Q 14

A small cone of height 8 cm is cut off from a bigger cone to leave a frustum of height 16cm. If the volume of the smaller cone is 160cm3, find the volume of the frustum (3marks)

Answer

L.S.F. = 8: 24 = 1: 3

V.S.F = 1: 27

Volume of frustum = $160 \times 27 - 160$

$= 4160\text{cm}^3$

PP1 Q 15

The production of milk, in litres, of 14 cows on a certain day was recorded as follows: 22, 26, 15, 19, 20, 16, 27, 15, 19, 22, 21, 20, 22 and 28.

Determine:

(a) the mode; (1 mark)

Answer

Mode

= 22

(b) the median. (2 marks)

Answer

Median

15, 15, 16, 19, 19, 20, 20, 21, 22, 22, 22, 26, 27, 28

$\text{median} = \frac{20 + 21}{2}$

= 20.5

P1 Q 16

Given that log 4=0.6021 and log 6=0.7782, without using mathematical tables or a calculator, evaluate log 0.096. (3mks)

Answer

$\text{Log } 0.096 = \text{Log}\left(4^2 \times 6 \times 10^{-3}\right)$

$= 2(0.6021) + \bar{3}.7782$

$= \bar{2}.9824 \text{or}$

(-1.0176)

P1 Q 17

(a) Solve the equation, $\frac{x+3}{24} = \frac{1}{x-1}$ ***(4 marks)***

Answer

X = 5 or - 6

(b) The length of a floor of a rectangular hall is 9 m more than its width. The area of a floor is 136 m2.

(i) Calculate the perimeter of the floor. (4 marks)

Answer

50 m

(ii) A rectangular carpet is placed on the hall leaving an area of 64 m2. If the length of the carpet is twice its width, determine the width of the carpet. (2 marks)

Answer

6 m

PP No.18.

Three business partners: Asha, Nangila and Cherop contributed Ksh 60 000, Ksh 85 000 and Ksh 105 000 respectively. They agreed to put 25% of the profit back into business each year. They also agreed to,put aside 40% of the remaining profit to cater for taxes and insurance. The rest of the profit would then be shared among the partners in the ratio of their contributions.

At the end of the first year, business realized a gross profit of Ksh 225 000

(a) Calculate the amount of money Cherop received more than Asha at the end of the first year. (5 marks)

Answer

% Profit for taxes and insurance

$$= \frac{40}{100} \times \frac{75}{100}$$

Amount shared

$$= \frac{100-(25+30)}{100} \times 225000$$

$$= \frac{45}{100} \times 225000$$

$$= 101250$$

Amount Cherop received more than Asha: Ratio of contribution

60000: 85000: 105000

12 : 17 : 21

$$\frac{21-12}{50} \times 101250$$

$$= 18225$$

(b) Nangila further invested Ksh 25 000 into the business at the beginning of the second year. Given that the gross profit at the end of the second year increased in the ratio 10:9, calculate Nangila's share of the profit at the end of the second year. (5 marks)

Answer

Profit during 2nd year:

$225000 \times \frac{10}{9}$

$= 250000$

Nangila's new ratio:

$= \frac{110000}{275000} = \frac{2}{5}$

∴ Nangila's New Share of Profit

$= \frac{2}{5} \times 112500$

$= 45000$

P1 Q19

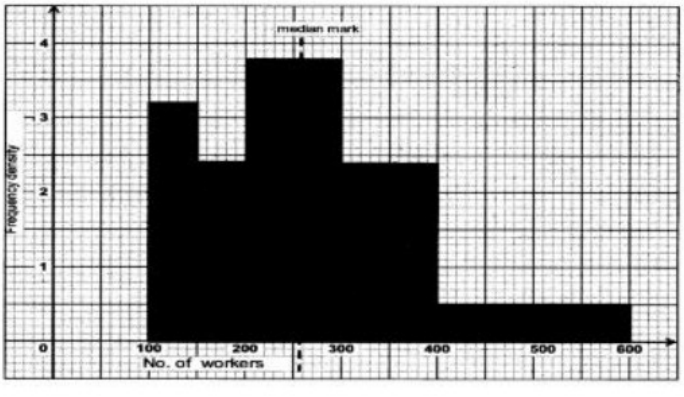
The frequency table below shows the daily wages paid to casual workers by a certain company

Wages in shillings	100-150	150-200	200-300	300-400	400-600
No. of workers	160	120	380	240	100

a) Draw a histogram to represent the above information. (5 marks)

Answer

Frequency (f)	160	120	380	240	100
Class width(c)	50	50	100	100	200
Freq. density(f/c)	3.2	2.4	3.8	2.4	0.5



(b)(i)State the class in which the median wage lies. (1 mark)

Answer

median class: 200 – 300

(ii)Draw a vertical line, in the histogram, showing where the median wage lies. (1 mark!

Answer

Shown by dotted line (257.895)

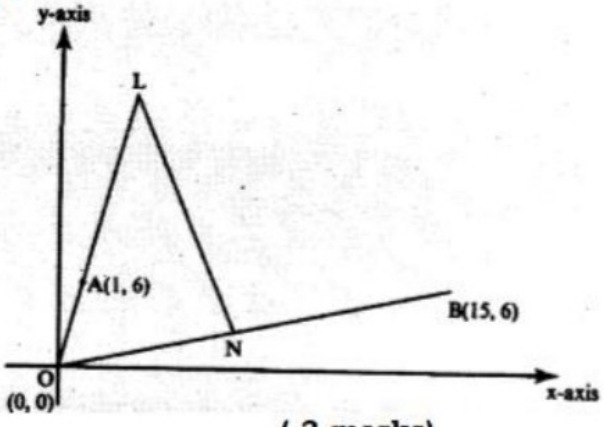
(c)Using the histogram, determine the number of workers who earn sh 450 or less per day. (3 mark;

Answer

Number of workers who earn Sh. 450 or less per day = $900 + 50 \times 0.5 = 925$

P1 Q 20

In the diagram below, the coordinates of points A and B are (1,6) and (15,6) respectively). Point N is on OB such that 3 ON = 2OB. Line OA is produced to L such that OL = 3 OA



a)Find vector LN (3 marks)

Answer

$$OL = 3\begin{pmatrix}1\\6\end{pmatrix}$$

$$= \begin{pmatrix}3\\18\end{pmatrix}$$

$$ON = \frac{2}{3}\begin{pmatrix}15\\6\end{pmatrix}$$

$$= \begin{pmatrix}10\\4\end{pmatrix}$$

$$LN = ON - OL$$

$$= \begin{pmatrix}10\\4\end{pmatrix} - \begin{pmatrix}3\\18\end{pmatrix}$$

$$= \begin{pmatrix}7\\-14\end{pmatrix}$$

(b)Given that a point M is on LN such that LM: MN = 3: 4, find the coordinates of (2 marks)

Answer

$$OM = OL + \frac{3}{7}LN$$
$$= \begin{pmatrix} 3 \\ 18 \end{pmatrix} + \frac{3}{7}\begin{pmatrix} 7 \\ -14 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ 18 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$
$$= \text{M}(6,12)$$

(c)If line OM is produced to T such that OM: MT = 6:1

(i) Find the position vector of T (1 mark)

Answer

$$OT = \frac{7}{6}OM$$
$$= \frac{7}{6}\begin{pmatrix} 6 \\ 12 \end{pmatrix}$$
$$= \begin{pmatrix} 7 \\ 14 \end{pmatrix}$$

(ii) Show that points L, T and B are collinear (4 marks)

Answer

$$LT = \begin{pmatrix} 7 \\ 14 \end{pmatrix} - \begin{pmatrix} 3 \\ 18 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$
$$LB = \begin{pmatrix} 15 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 18 \end{pmatrix}$$
$$= \begin{pmatrix} 12 \\ -12 \end{pmatrix}$$
$$LB = 3LT$$

L is the common point.

P1 Q 21

(a) The ratio of Juma's and Akinyi's earnings was 5 : 3. Juma's earnings rose to Ksh 8400 after an increase of 12%. Calculate the percentage increase in Akinyi's earnings given that the sum of their new earnings was Ksh 14 100. (6 marks)

Answer

Juma's earnings before increase:

$112\% \rightarrow 8400$

$100\% \rightarrow 8400 \times \frac{100}{112}$

$= 7500$

Akinyi's earnings before increase:

$\frac{3}{5} \times 7500 = 4500$

Increase in Akinyi's earnings

$14100 - 8400 - 4500$

$= 1200$

% increase in Akinyi's earnings

$\frac{1200}{4500} \times 100$

$= 26\frac{2}{3}$

(b) Juma and Akinyi contributed all the new earnings to buy maize at Ksh 1175 per bag. The maize was then sold at Ksh 1 762.50 per bag. The two shared all the money from the sales of the maize in the ratio of their contributions. Calculate the amount that Akinyi got. (4 marks)

Answer

No. of bags bought

$\frac{14100}{1175}$

$= 12$ bags

$\text{Profit} = (1762.50 - 1175) \times 12 = 7050$

Ratio 5700:8400 = 19:28

$\text{Profit for Akinyi:} = 7050 \times \frac{19}{47} = 2850$

Total earning for Akinyi:

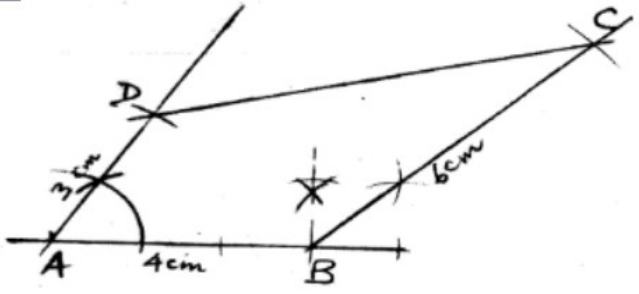
$5700 + 2850$

$= 8550$

P1 Q 22

Using a pair of compasses and ruler only, construct a quadrilateral ABCD in which AB = 4 cm, BC = 6 cm, AD = 3 cm, angle ABC = 135° and angle DAB = 60°. Measure the size of angle BCD. (4 marks)

Answer



Construction of 135^0 angle between lines AB = 4 cm and BC = 6 cm

Construction of 60^0 angle between lines AB = 4 cm and AD = 3 cm

Completion of quadrilateral ABCD

$\angle BCD = 31° \pm 1°$

P1 Q 23

The equation of a curve is $y = 2x^3 + 3x^2$.

(a) Find:

(i) The x-intercept of the curve; (2 marks)

Answer

x - intercepts

when y= 0

$x^2(2x + 3) = 0$

$x = 0$ *and* $x = -\frac{3}{2}$

(ii) The y- intercept of the curve; (1 mark)

Answer

y - intercept

when x = 0, y = 0

(b)(i)Determine the stationary points of the curve.(3 marks)

Answer

stationary points of curve

$\frac{dy}{dx} = 6x^2 + 6x$

stationery points when $\frac{dy}{dx} = 0$

i.e. $6x^2 + 6x = 0$

$6x(x+1) = 0$

$x = 0$ *or* $x = -1$

$\therefore$ stationary points are:

$(0,0)$ and $(-1,1)$

(ii) For each points in (b) (i) above, determine whether it is a maximum or minimum, (3 marks)

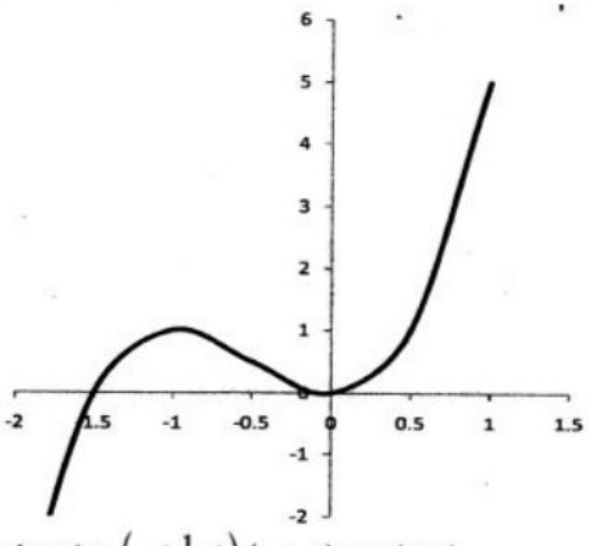
Answer

x	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$\frac{dy}{dx}$	12	$4\frac{1}{2}$	0	$-1\frac{1}{2}$	0	$4\frac{1}{2}$	12

minimum point (0,0)
maximum point (-1,1)

(c) Sketch the curve. (2 marks)

Answer



points plotted at $\left(-1\frac{1}{2},0\right),(-1,1)$ *and* $(0,0)$

smooth curve

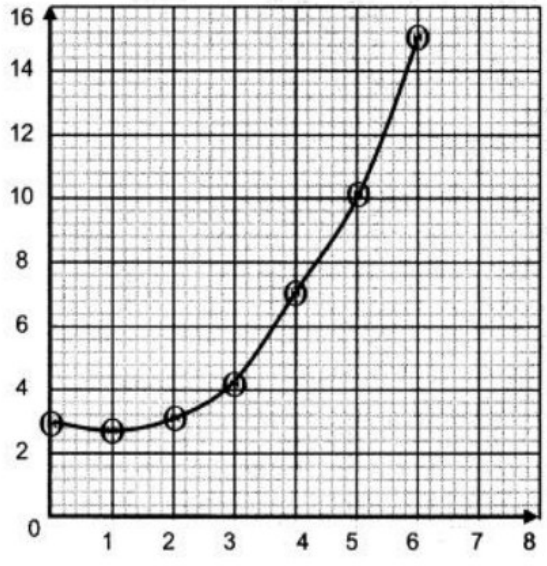
P1 Q 24

(a)On the grid provided, draw a graph of the function $y = \frac{1}{2}x^2 - x + 3$ for $0 \leq x \leq 6$

(grid was provided) (3 marks)

Answer

x	0	1	2	3	4	5	6
$y = \frac{1}{2}x^2 - x + 3$	3	2½	3	4½	7	10½	15



PAPER 2

P2 Q 1

Simplify $\frac{4}{\sqrt{5}+\sqrt{2}} - \frac{3}{\sqrt{5}-\sqrt{2}}$ **(3 marks)**

Answer

$$\frac{4}{\sqrt{5}+\sqrt{2}} - \frac{3}{\sqrt{5}-\sqrt{2}}$$
$$= \frac{4(\sqrt{5}-\sqrt{2}) - 3\sqrt{5}+\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$
$$= \frac{4\sqrt{5}-4\sqrt{2}-3\sqrt{5}-3\sqrt{2}}{3}$$
$$= \frac{\sqrt{5}-7\sqrt{2}}{3}$$

P2 Q 2

By correcting each number to one significant figure, approximate the value of 788x0.006. Hence calculate the percentage error arising from this approximation. (3 marks)

Answer

Writing each number to one significant figure: $788 \approx 800$ and $0.006 = 0.006$

Thus, $800 \times 0.006 = 4.8$

$$\% \text{ error} = \frac{4.8 - (788 \times 0.006)}{788 \times 0.006} \times 100\ \%$$
$$= \frac{0.072}{4.728} \times 100\%$$
$$= 1.523\% \text{ or } (1.52284264\%)$$

P2 Q 3

The area of triangle FGH is 21cm². The triangle FGH is transformed using the matrix

$\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$ **Calculate the area of the image of triangle FGH (2 Marks)**

Answer

Image area $[(4\times 2)-(5\times 1)]\times 21$

63 sq. units

P2 Q 4

Make s the subject of the formula.

$$w = \sqrt[3]{\frac{s+t}{s}}$$

Answer

$$w^3 = \frac{s+t}{s}$$

$$w^3 s = s + t$$

$$w^3 s - s = t$$

$$s = \frac{t}{w^3 - 1}$$

P2 Q 5

Solve the equation 2log x – log(x-2)= 2log 3 (3mks)

Answer

$$Log\left(\frac{x^2}{(x-2)}\right) = \log 3^2$$

$$\frac{x^2}{x-2} = 9$$

$$x^2 - 9x + 18 = 0$$

$$(x - 6)(x - 3) = 0$$

$$x = 6 \text{ or } x = 3$$

P2 Q 6

Kago deposited Ksh 30 000 in a financial institution that paid simple interest at the rate of 12% per annum. Nekesa deposited the same amount of money as Kago in another financial institution that paid compound interest. After 5 years, they had equal amounts of money in the financial institutions.
Determine the compound interest rate per annum, to 1 decimal place, for Nekesa's deposit. (4 marks)

Answer

Amount for Kago

$$= 30000 + \frac{12}{100} \times 30000 \times 5$$

$$= 48000$$

Compound interest rate for Nekesa

$$30000\left(1 + \frac{r}{100}\right)^5 = 48000$$

$$\left(1 + \frac{r}{100}\right)^5 = \frac{48000}{30000} = 1.6$$

$$1 + \frac{r}{100} = \sqrt[5]{1.6}$$

$$r = 100(1.098560543 - 1)$$

$$= 9.9\%$$

P2 Q7

The masses in kilograms of 20 bags of maize were; 90,94,96,98,99,102,105 91 102 99 105,94,99,90,94,99,98,96,102 and 105.
Using an assumed mean of 96kg, calculate the mean mass, per bag, of the maize. (3 marks)

Answer

Differences from assumed mean

$$-6-2+0+2+3+6+9-5+6+3+9$$
$$-2+3-6-2+3+2+0+6+9=38$$

$$\therefore mean = 96+\frac{38}{20}$$
$$= 97.9$$

P2 Q 8

The first term of an arithmetic sequence is - 7 and the common difference is 3.

(a)List the first six terms of the sequence; (1 mark)

Answer

First 6 terms

-7, -4, -1, 2, 5, 8

(b)Determine the sum of the first 50 terms of the sequence. (2 marks)

Answer

Sum of 1st 50 terms

$$S_{50}=\frac{50}{2}\{2\times-7+49\times3\}$$

$$= 3325$$

P2 Q 9

A bag contains 2 white balls and 3 black balls. A second bag contains 3 white balls and 2 black balls. The balls are identical except for the colours.
Two balls are drawn at random, one after the other from the first bag and placed in the second bag. Calculate the probability that the 2 balls are both white. (2 marks)

Answer

$$P(ww)=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$$

$$P(bb)=\frac{2}{7}\times\frac{1}{6}=\frac{1}{21}$$

P2 Q. 10

An arc 11 cm long, subtends an angle of 70° at the centre of a circle. Calculate the length, correct to one decimal place, of a chord that subtends an angle of 90° at the centre of the same circle. (4 marks)

Answer

Length 12.7 cm

P2 Q 11

Given that q $\mathbf{i}$ + $^1/_3$ $\mathbf{j}$ + $^2/_3$ $\mathbf{k}$ is a unit vector, find q (2 marks)

Answer

$$q^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2=1^2$$

$$q^2=1-\frac{5}{9}=\frac{4}{9}$$

$$q=\pm\frac{2}{3}$$

P2 Q 12

(a) Expand the expression $(1+\frac{1}{2}x)^5$ in ascending powers of x, leaving the coefficients as fractions in their simplest form (2 marks)

Answer

$$1.1^5(\tfrac{1}{2}x)^0+5.1^4(\tfrac{1}{2}x)^1+10.1^3(\tfrac{1}{2}x)^2+10.1^2(\tfrac{1}{2}x)^3+5.1(\tfrac{1}{2}x)^4+1.1^0(\tfrac{1}{2}x)^5$$

$$=1+\tfrac{5}{2}x+\tfrac{5}{2}x^2+\tfrac{5}{4}x^3+\tfrac{5}{16}x^4+\tfrac{1}{32}x^5$$

(b) Use the first three terms of the expansion in (a) above to estimate the value of $(1\frac{1}{20})^5$ (2 marks)

Answer

$$(1\tfrac{1}{20})^5\approx 1+\tfrac{5}{2}\times\tfrac{1}{10}+\tfrac{5}{2}\times\tfrac{1}{100}$$

$$=1+\tfrac{5}{20}+\tfrac{5}{200}$$

$$=1\tfrac{11}{40}$$

P2 Q 13

A circle whose equation is $(x-1)^2 + (y-k)^2 = 10$ passes through the point (2, 5), Find the coordinates of the two possible centres of the circle,

Answer

$$(2-1)^2 + (5-k)^2 = 10$$
$$k^2 - 10k + 16 = 0$$
$$(k-2)(k-8) = 0$$
$$k = 2 \text{ or } k = 8$$

Centre at $(1,2)$ or $(1,8)$

P2 Q 14

Water and milk are mixed such that the ratio of the volume of water to that of milk is 4: 1. Taking the density of water as 1 g/cm3 and that of milk as 1.2g/cm3, find the mass in grams of 2.5 litres of the mixture. (3 marks)

Answer

Vol. of water = $\frac{4}{5} \times 2500 = 2000\ cm^3$

Vol. of milk = $\frac{1}{5} \times 2500 = 500\ cm^3$

The mass of the mixture = $2000 \times 1g + 500 \times 1.2g$
$= 2600g$

PP2 No. 15]

A school decided to buy at least 32 bags of maize and beans. The number of bags of maize were to be more than 20 and the number of bags of beans were to be at least 6. A bag of maize costs Ksh 2500 and a bag of beans costs Ksh 3500. The school had Ksh 100 000 to purchase the maize and beans. Write down all the inequalities that satisfy the above information. (4 marks)

Answer

$$x+y \geq 32, x > 20, y \geq 6, 5x + 7y \leq 200$$

P2 Q 16

Find in radians, the values of x in the interval $0^c \leq x \leq 2\pi^c$ for which $2\cos^2 x - \sin x = 1$. (Leave the answers in terms of π) (4mks)

Answer

$$2(1-\sin^2 x) - \sin x = 1$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$2\sin^2 x + 2\sin x - \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$\sin x = \frac{1}{2} \text{ or } \sin x = -1$$
$$x = \frac{1}{6}\pi^c, \frac{5}{6}\pi^c, \frac{3}{2}\pi^c$$

(b) Calculate the mid-ordinates for 5 strips between x = 1 and x = 6, and hence use the mid-ordinate rule to approximate the area under the curve between x = 1, x = 6 and the x-axis. (3 marks)

Answer

$$y_1 = \frac{1}{2}\times 1.5^2 - 1.5 + 3 = 2.625$$

$$y_2 = \frac{1}{2}\times 2.5^2 - 2.5 + 3 = 3.625$$

$$y_3 = \frac{1}{2}\times 3.5^2 - 3.5 + 3 = 5.625$$

$$y_4 = \frac{1}{2}\times 4.5^2 - 4.5 + 3 = 8.625$$

$$y_5 = \frac{1}{2}\times 5.5^2 - 5.5 + 3 = 12.625$$

Approximate area

$$= 1(2.625 + 3.625 + 5.625 + 8.625 + 12.625)$$

$$= 33.125 \text{ sq. units}$$

(c) Assuming that the area determined by integration to be the actual area, calculate the percentage error in using the mid-ordinate rule. (4 marks)

Answer

$$\text{Area} = \int_1^6 \left(\frac{1}{2}x^2 - x + 3\right)dx = \left[\frac{x^3}{6} - \frac{x^2}{2} + 3x\right]_1^6$$

$$= \left[\frac{6^3}{6} - \frac{6^2}{2} + 3\times 6\right] - \left[\frac{1^3}{6} - \frac{1^2}{2} + 3\right] = 33.5$$

$$\%\text{ error} = \frac{33.5 - 33.125}{33.5}\times 100$$

$$= 0.625\%$$

P2 Q 17

A garden measures 10 m long and 8 m wide. A path of uniform width is made all round the garden. The total area of the garden and the path is 168 m2.

(a) Find the width of the path. (4 marks)

Answer

Let width of the path be x

$$\text{Area} = (10 + 2x)(8 + 2x) = 168$$

$$\Leftrightarrow 80 + 20x + 16x + 4x^2 = 168$$

$$4x^2 + 36x - 88 = 0$$

$$\Leftrightarrow x^2 + 9x - 22 = 0$$

$$(x - 2)(x + 11) = 0$$

$$x = 2 \text{ or } -11$$

Width of path = 2m

(b) The path is to be covered with square concrete slabs. Each corner of the path is _ covered with a slab whose side is equal to the width of the path.
The rest of the path is covered with slabs of side 50 cm. The cost of making each corner slab is sh 600 while the cost of making each smaller slab is sh 50.
Calculate:

(i) the number of the smaller slabs used. (3 marks)

Answer

Area covered by small slabs

$$= \frac{14}{68} \times 12 - (10 \times 8 + 4(2 \times 2))$$

$$\text{No. of slabs} = \frac{72}{0.5 \times 0.5}$$

$$= 288$$

(ii) the total cost of the slabs used to cover the whole path. (3 marks)

Answer

Cost of slabs

Large $= 600 \times 4$

$= 2400$

Small $= 50 \times 288$

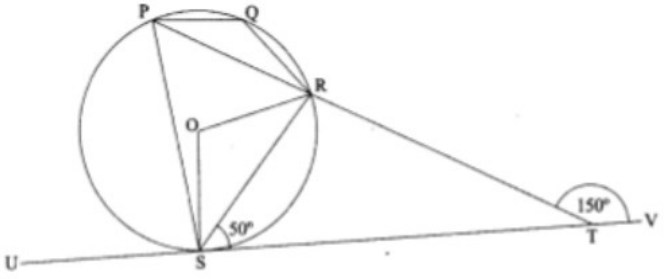
$=14400$

Total cost = 2400+14400

= sh16, 800

P2 Q 18

In the figure below, P, Q, R and S are points on the circle centre O. PRT and USTV are straight lines. Line USTV is a tangent to the circle at S, ∠RST = 50° and ∠RTV = 150°.



a) Calculate the size of:

(i) ∠ORS; (2 marks)

Answer

$$\angle OSR = 90 - 50 = 40$$

$$\therefore ORS = 40$$

(ii) ∠USP; (1 mark)

Answer

$$\angle RTS = 30^0 \text{ and } \angle RPS = 50^0$$

$$\therefore \angle USP = 30^0 + 50^0 = 80^0$$

(iii) ∠PQR. (2 marks)

Answer

$$\angle PQR = 180^0 - 50^0 = 130^0$$

(b) Given that RT = 7 cm and ST = 9 cm, calculate to 3 significant figures:

(i) the length of line PR; (2 marks)

Answer

$$PT \times TR = TS^2$$

$$(7 + x)(7) = 9^2$$

$$7x = 81 - 49 = 32$$

$$x = \frac{32}{7} = 4.57$$

(ii) the radius of the circle. (3 marks)

Answer

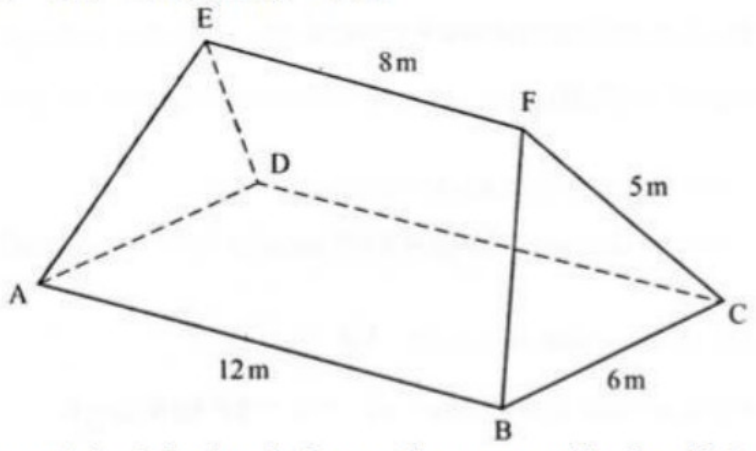
$$\angle ORP = 40^0$$

$$\cos 40^0 = \frac{\frac{1}{2} \times 4.57}{r}$$

$$r = \frac{\frac{1}{2} \times 4.57}{Cos\ 40} = 2.98$$

P2 Q 19

The figure ABCDEF below represents a roof of' a house. AB = DC = 12 m, BC = AD = 6 m, AE = BF = CF = DE = 5m and EF = 8m.



(a) Calculate, correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

Answer

(a) = distance of EF from place ABCD

slant height from F to BC

$= \sqrt{5^2 - 3^2}$

$= 4$

$\therefore$ = distance of EF from plane ABCD

$= \sqrt{4^2 - 2^2}$

$= \sqrt{12} = 3.46$ m

(b) Calculate the angle between:

(i) the planes ADE and ABCD; (2 marks)

Answer

(i) angle between planes

ADE and ABCD

$= \tan^{-1} \frac{\sqrt{12}}{2}$

$= 60°$

(ii) the line AE and the plane ABCD, correct to 1 decimal place; (2 marks)

Answer

angle between line AE and plane ABCD

$= \sin^{-1}\frac{\sqrt{12}}{5}$

$= 43.9°$

(iii) the planes ABFE and DCFE, correct to 1 decimal place. (3 marks)

Answer

angle between planes

ABFE and DCFE

$= 2c\tan^{-1}\frac{3}{\sqrt{12}}m$

$= 81.8°$

P2 Q 20

A water vendor has a tank of capacity 18 900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000cm^3/minute through pipe A and 120 000 cm ^{3}/ minute through pipe B.

a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank (3 marks)

Answer

Total rate of flow in litres

120 + 150 = 270 l/min

Time taken = $\frac{18900}{270}$

= 70 min (1 hr 10 min)

b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

i) Calculate the time it took to fill the tank on that day (4 marks)

Answer

Part of tank filled after 25 min

270 x 25

= 6750

Time taken to fill remaining part

$= \frac{18900 - 6750}{270 - 20}$

= 48.6 min

Total time to fill tank

= 25 + 48.6 = 73.6 min

ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6300 litres, calculate, in litres, the amount of water that was wasted (3 marks)

Answer

Total inflow into tank
= 270 x 73.6
= 19872
Water wasted = 91872 – (542 x 25 + 6300)
= 22 l

P2 Q 21

A tourist took lh 20minutes to travel by an aircraft from town T(3°S, 35°E) to town U(9°N,35°E).

(Take the radius of the earth to be 6370km and ($\pi = \frac{22}{7}$)

(a) Find the average speed of the aircraft.(3 marks)

Answer

distance from T to U

$$= 2 \times 6370 \times \frac{22}{7} \times \frac{12}{360}$$

$$\text{speed} = \frac{2 \times 6370 \times \frac{22}{7} \times \frac{12}{360}}{1\frac{1}{3}}$$

$$= 1001 \text{ km/h}$$

(b)After staying at town U for 30 minutes, the tourist took a second aircraft to town V(9°N, 5°E). The average speed of the second aircraft was 90% that of the first
aircraft. Determine the time, to the nearest minute, the aircraft took to travel from U to V. (3 marks)

Answer

$$\text{time} = \frac{2 \times 6370 \times \frac{22}{7} \times \frac{30}{360} \cos 9^\circ}{1001 \times \frac{90}{100}}$$

$$= 3.658104965 \text{ h}$$
$$\simeq 3 \text{ h } 39 \text{ min}$$

(c) When the journey started at town T, the local time was 0700h. Find the loca time at V when the tourist arrived. (4 marks)

Answer

Arrival time at U
0700 + 1h 20 min
= 0820 h

Departure time at U
0820 + 30 min
= 0850 h

Time difference between U and V
$\frac{35-5}{360} \times 24$

= 2h

Arrival time at V (local time)
0850h + 3h 39min - 2h
= 1029h

P2 Q 22

The gradient function of a curve is given by the expression 2x + 1. If the curve passes through the point (-4, 6);

(a) Find:

(i) The equation of the curve (3 marks)

Answer

$y = \frac{2x^2}{2} + x + c$

at $x = -4$, $y = 6$

$6 = (-4)^2 - 4 + c$

$c = -6$

$y = x^2 + x - 6$

(ii) The values of x, at which the curve cuts the x- axis (3 marks)

Answer

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

$x = 2$ or $x = -3$

(b) Determine the area enclosed by the curve and the x- axis (4 marks)

Answer

$\int_{-3}^{2} (x^2 + x - 6)dx = \left[\frac{x^3}{3} + \frac{x^2}{2} - 6x\right]_{-3}^{2}$

$\left[\frac{8}{3} + \frac{4}{2} - 12\right] - \left[\frac{-27}{3} + \frac{9}{2} + 18\right]$

$-7\frac{1}{3} - 13\frac{1}{2} = -20\frac{5}{6}$

Area $= 20\frac{5}{6}$

P2 Q 23

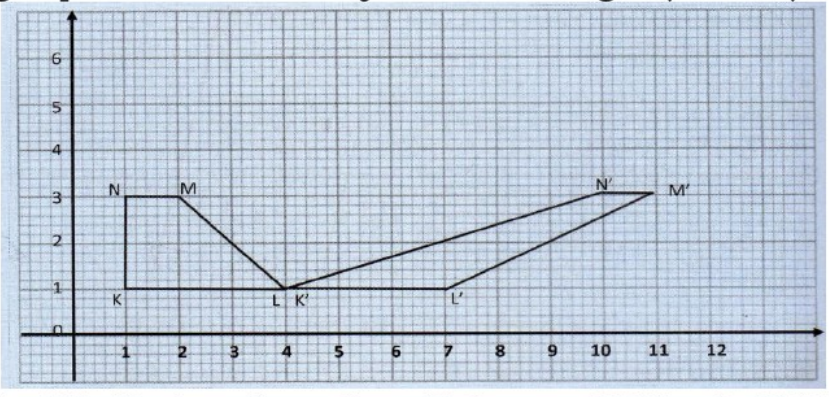
A quadrilateral with vertices at K (1, 1), L (4, 1), M (2, 3) and N (1, 3) is transformed by a matrix $T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ ***to a quadrilateral K'L'M'N'***

(a)Determine the coordinates of the image. (3 marks)

Answer

K! (4,1) (7,1),M! (11,3) N! (10,3)

(b)On the grid provided draw the object and the image. (2 marks)



(c)(i) Describe fully the transformation which maps KLMN onto K'L'M'N. (2 marks)

Answer

Shear parallel to x-axis of shear factor 3 with the x-axis as the invariant line

(ii) Determine the area of the image. (1 mark)

Answer

Area of image = Area of object = 4 sq units

(d) Find a matrix which maps K'L'M'N' onto KLMN. (2 marks)

Answer

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

P2 Q 24

The first, fifth and seventh terms of an arithmetic progression (AP) correspond to the first three consecutive terms of a decreasing Geometric Progression (GP).The first term of each progression is 64, and the common difference of the AP is d and the common ratio of the G.P is r.

a) (i) Write two equations involving d and r (2marks)

Answer

$64 + 4d = 64r$

$64 + 6d = 64r^2$

(ii) Find the values of d and r

Answer

From (i)

$d = 16r - 16$

$64r^2 = 64 + 6(16r - 16)$

$64r^2 = 64 + 96r - 96$

$2r^2 - 3r + 1 = 0$

$(2r - 1)(r - 1) = 0$

$r = ½$ or $r = 1$

For decreasing GP, $r = ½$

Substituting $r = ½$ in (i)

$64 \times ½ = 64 + 4d$

$d = -8$

b) Find the sum of the first 10 term of;

(i) the A.P

Answer

A.P

$$S_{10} = \frac{10}{2}\{2 \times 64 + 9 \times -8\} = 280$$

(ii) the G.P

Answer

G.P

$$S_{10} = \frac{64\left(1 - \frac{1}{2^{10}}\right)}{1 - \frac{1}{2}} = \frac{64\left(\frac{1023}{1024}\right)}{\frac{1}{2}} = 127.875$$