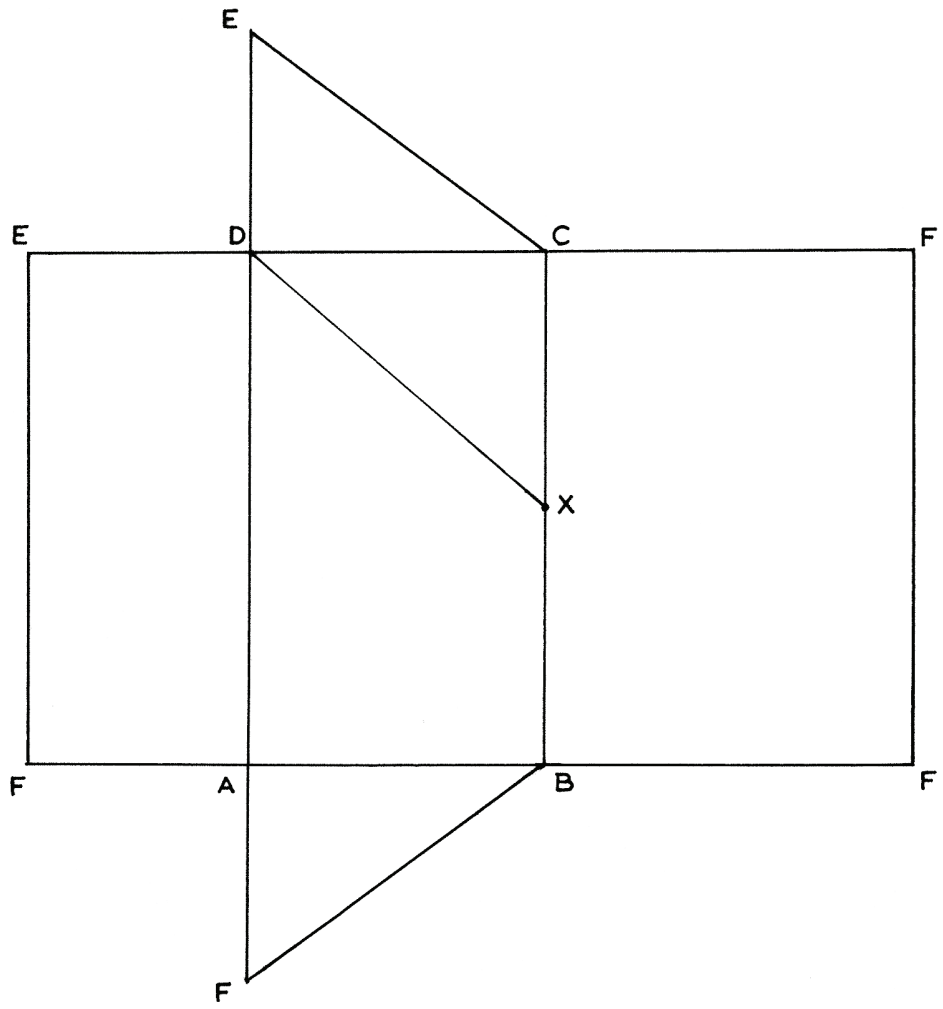
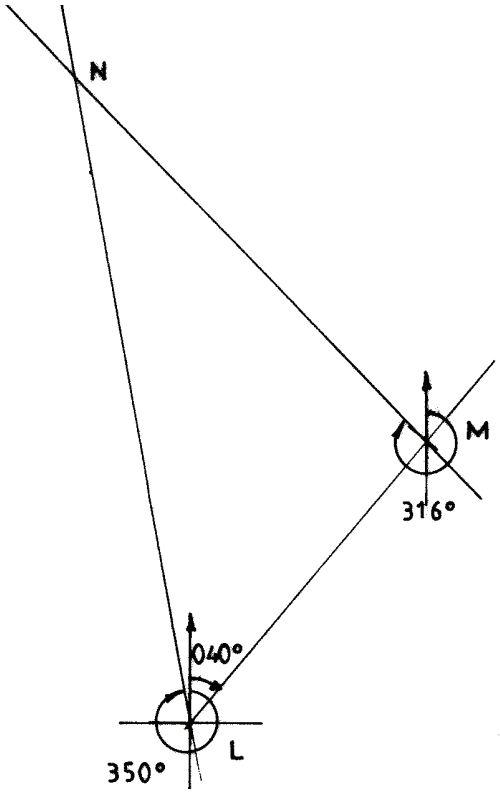
4.3 MATHEMATICS (121 AND 122)

4.3.1 Mathematics Alternative A Paper 1 (121/1

1.	Cows $= 32$ Sheep $= 32 \times 12$ $= 384$ Goats $= 384 + 1344$ $= 1728$ Number of goats that remained $= \frac{1}{4} \times 1728$ $= 432$	 M1 M1 M1 A1 4	
2.	$\frac{\sqrt{1764}}{\sqrt[3]{2744}} = \frac{\sqrt{2^2 \times 3^2 \times 7^2}}{\sqrt[3]{2^3 \times 7^3}}$ $= \frac{2 \times 3 \times 7}{2 \times 7}$ $= 3$	M1 M1 A1 3	For prime factors of both $\sqrt{\ }$ and $\sqrt[3]{\ }$
3.	Volume $= \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times 18$ $= 3696\,\text{cm}^3$ Density $= \frac{4.62 \times 1000}{3696}$ $= 1.25\,\text{g/cm}^3$	M1 M1 A1 3	

4.	 DX $= 5.3 \pm 0.1$	B1 B1 B1 3	√ measurements and angles √ complete net (labelled)
5.	C.P. for carpet $= \frac{36000 \times 100}{120}$ $= 30000$ % profit made during trade fair $= \frac{33600 - 30000}{30000} \times 100$ $= 12\%$	M1 M1 A1 3	

6.	$= \frac{243^{\frac{-2}{5}} \times 125^{\frac{2}{3}}}{9^{\frac{-3}{2}}}$ $= \frac{27 \times 25}{9}$ $= 75$	M1 M1 A1 3	√ manipulation of all indices or equivalent simplification
7.	$= \frac{\theta}{2\pi} \times \pi \times 2.1 \times 2.1 = 2.31$ $\theta = \frac{2.31 \times 2}{2.1 \times 2.1}$ $= 1.05^{c}$	M1 A1 2	
8.	$(x + 2y)^2 - (2y - 3)^2$ $= (x^2 + 4xy + 4y^2) - (4y^2 - 12y + 9)$ $= x^2 + 4xy + 12y - 9$	M1 A1 2	

9.		B1 B1	√ location of M √ location of N
	Distance MN = 6.8 × 100 = 680 km	M1 A1 4	MN = 6.8 ± 0.1 cm
10.	$(2n-4)\times 90 = 1800$ $180n = 2160$ $n = 12$ size of each exterior angle $= \frac{360}{12} = 30^\circ$	M1 M1 A1 3	
11.	let age of cow be x years $\therefore x\left(x - 4\frac{2}{3}\right) = 8$ $3x^2 - 14x - 24 = 0$ $(3x+4)(x-6) = 0$ $x = 6$ or $-\frac{4}{3}$ Age of cow = 6 years Age of heifer = $1\frac{1}{3}$ years	M1 M1 A1 B1 4	

12.	$4 \le 3x - 2 < 9 + x$ $4 \le 3x - 2 \quad 3x - 2 < 9 + x$ $6 \le 3x \quad 2x < 11$ $x \ge 2 \quad x < 5\frac{1}{2}$ $\therefore 2 \le x < 5\frac{1}{2}$ Integral values 2, 3, 4, 5	M1 A1 B1 3	
13.	Volume of water in container $= \frac{80}{100} \times 90(40 \times 25 - \pi \times 7.5^2)$ $= 59276.54975$ $\frac{59276.54975}{1000}$ $= 59.3$	 M1 M1 M1 A1 3	 for $\frac{80}{100} \times 90$ difference in volumes conversion into litres
14.	Angle for major arc $= 360 - 105$ $= 255^\circ$ Length of arc $= \frac{255}{360} \times 2 \times 8.4 \times \frac{22}{7}$ $= 37.4$ cm	 B1 M1 A1 3	
15.	Amount of work $= 25 \times 16 \times 9$ Machines required $= \frac{25 \times 16 \times 9}{12 \times 10}$ $= 30$	M1 M1 A1 3	 $\div$ by 12×10
16.	$\lvert AB \rvert = \sqrt{(-3+2)^2 + (7-2)^2} = \sqrt{26}$ $\lvert A'B' \rvert = \sqrt{4^2 + (-20)^2} = \sqrt{416}$ Scale factor $= \frac{\lvert A'B' \rvert}{\lvert AB \rvert} = \frac{\sqrt{416}}{\sqrt{26}}$ $= 4$	 M1 M1 A1 3	 for $\lvert AB \rvert$ and $\lvert A'B' \rvert$

17.	(a) Equation of L		
	gradient $= \frac{6-3}{-1-^{-}2}$	M1	
	$= 3$		
	equation $= \frac{y-6}{x+1} = 3$		
	$\Rightarrow y - 3x = 9$	A1	
	(b) equation of P		
	$= \frac{y-6}{x+1} = -\frac{1}{3}$	M1	
	$3y + x = 17$	A1	
	(c) equation of Q		
	$= \frac{y-2}{x-1} = 3$		
	$y = 3x - 1$	B1	
	x intercept when $y = 0 \Rightarrow x = \frac{1}{3}$	B1	
	y intercept when $x = 0 \Rightarrow y = -1$	B1	
	(d) Intersection of lines P and Q		
	$3y + x = 17..(i)$ $y - 3x = -1..(ii)$	M1	
	$3y + x = 17$ $3y - 9x = -3$ $10x = 20 \Rightarrow x = 2$	A1	for both $x = 2$ and $y = 5$
	subset $3y + 2 = 17 \Rightarrow y = 5$ $\therefore$ point of intersection $(2, 5)$	B1	
		10	

18. (a)

Class	3-5	6-8	9-11	12-14	15-17	18-20
Frequency	3	8	13	10	4	2

B1
B1

(b) (i) mean length $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{4 \times 3 + 7 \times 8 + 10 \times 13 + 13 \times 10 + 16 \times 4 + 19 \times 2}{40}$ B1 for all √ mid points - i.e 4, 7, 10, 13, 16, and 19

M1

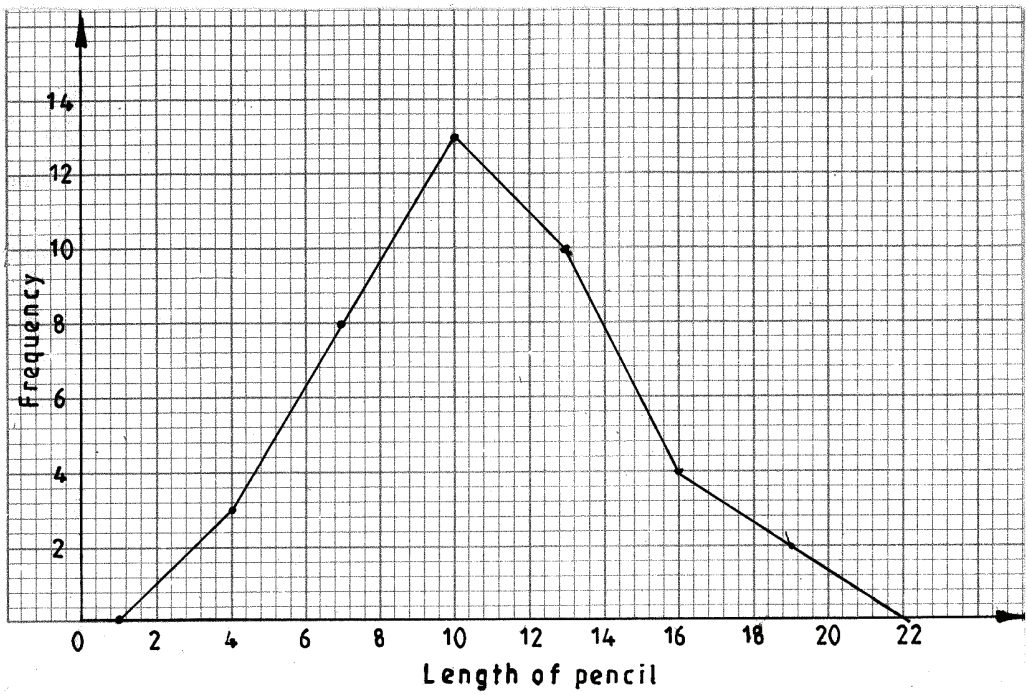
$= 10.75$ A1

(ii)

$= \frac{23}{40} \times 100$ B1 for 23

$= 57.5\%$ B1

(c)



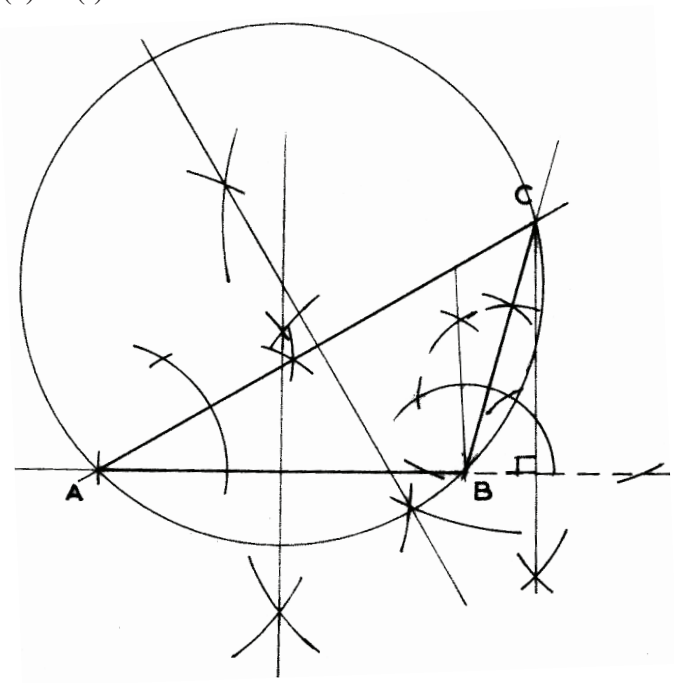
S1
P1
C1

10

19.	(a)	15 m/s	B1	
	(b)	maximum speed		
		$\frac{1}{2}(15+h)\times 10+\frac{1}{2}(10+30)h=825$	M1	
		$75+5h+20h=825$	M1	
		$25h=750$		
		$h=30$ m/s	A1	
	(c) (i)	$=\frac{30-15}{10}$	M1	
		$=1.5$ m/s^2	A1	
	(ii)	$=\frac{0-30}{20}=-1.5$ m/s^2	B1	
	(d)	$\left[\frac{1}{2}(15+30)\times 10+10\times 30\right]\div 20$	M1	for distance covered in first 20 seconds
		$=(225+300)\div 20$	M1	
		$=26.25$ m/s	B1	
			10	

20.	(a) base area $= \frac{1}{2} \times 15 \times 15 \sin 72 \times 5$	B1 M1	use of 72^o
	$= 534.97$	A1	
	(b) Length AV $= \sqrt{36^2 + 15^2} = 39$	B1	
	(c) Area of triangular faces: $\frac{AB}{\sin 72} = \frac{15}{\sin 54}$		
	$AB = \frac{15 \sin 72}{\sin 54}$	M1	
	$= 17.63$		
	$\therefore$ area $= \sqrt{\left\{\frac{1}{2}(39 + 39 + 17.63)(30.185)(8.815^2)\right\}}$	M1	$\surd$ application of Herons formula
	$= 334.89$		
		M1 A1	
	Total area $= 334.89 \times 5 + 534.97$ $= 2209.42$		
	(d) volume of pyramid $= \frac{1}{3} \times 534.97 \times 36$	M1 A1	
	$= 6419.63 \text{ cm}^2$ $\simeq 6420$ (4 s.f.)	10	

21.	(a) $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|c\|c\|c\|c\|c\|}\hline x & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline y & 16 & 10 & 6 & 4 & 4 & 6 & 10 & 16 & 24 & 34 & 46 \\ \hline \end{array}$	B2	y values (B1 for at least 6 correct)
	(b) Area using trapezium rule $= \frac{1}{2} \times 1\left[16 + 46 + 2\left(10 + 6 + 4 + 4 + 6 + 10 + 16 + 24 + 34\right)\right]$	M1	
	$= \frac{1}{2}[62 + 2(114)]$	M1	simplification
	$= 145$	A1	
	(c) Area using mid-ordinate rule		
	$= 2 \times (10 + 4 + 6 + 16 + 34)$	M1	
	$= 140$	A1	
	(d) Area using integration method		
	$\int_{-2}^{8} (x^2 - 3x + 6)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 6x \Big]_{-2}^{8}$	M1	√ integration
	$= \left[\frac{512}{3} - \frac{192}{2} + 48\right] - \left[\frac{-8}{3} - \frac{3 \times 4}{2} - 12\right]$	M1	
	$= 122\frac{2}{3} + 20\frac{2}{3}$	A1	
	$= 143\frac{1}{3}$	10	

22.	(a) (i) 	B1 B1 B1	construction of 30° construction of 105° completion of △ABC
	(ii)	B1	⊥ bisectors
		B1	circle
	radius = 3.5 ± 0.1	B1	
	(iii) height construction	B1	height constructed
	height = 3.4 ± 0.1	B1	
	(b) area of circle outside triangle $= \pi \times 3.5^2 - \frac{1}{2} \times 3.4 \times 5$	M1	
	$\simeq 29.98$	A1	
		10	

23.	(a) $\tan\theta = \frac{70}{240}$	M1	
	$= 0.2917$		
	$\theta = 16.26^\circ$	A1	
	(b) AC $= \sqrt{70^2 + 240^2}$		
	$= 250$ m	B1	
	$\angle$ ACD $= 150^\circ - (90^\circ - 16.26^\circ)$	M1	
	$= 76.26^\circ$		
	$AD^2 = 200^2 + 250^2 - 2 \times 200 \times 250 \cos 76.26$	M1	
	AD $= \sqrt{40000 + 62500 - 100000 \cos 76.26^\circ}$		
	$= 280.6$	A1	
	(c) Area of plot		
	$= \frac{1}{2} \times 240 \times 70 + \frac{1}{2} \times 250 \times 200 \sin 76.26^\circ$	M1	
	$= 8400 + 24284.59$	M1	
	$= 32684.59$ m^2		
	$= \frac{32684.59}{10000}$	M1	
	$= 3.27$ ha	A1	
		10	

24.	(a) Value of y when $x=-1$ $y=-1-4+3=-2$	B1	
	(b) Stationary points $\frac{dy}{dx}=3x^2-8x-3$	M1	
	for stationary points $3x^2-8x-3=0$ $(3x+1)(x-3)=0$	M1	
	$x=-\frac{1}{3}$ or $x=3$	A1	
	when $x=-\frac{1}{3}$, $y=\frac{14}{27}$ when $x=3$, $y=-18$		
	$\therefore$ stationary points $\left(-\frac{1}{3},\frac{14}{27}\right)$	B1	
	and (3, -18)	B1	
	(c) Equation of normal to curve: gradient of tangent at $x=1$		
	$\frac{dy}{dx}=3-8-3=-8$	B1	
	gradient of normal $=\frac{1}{8}$	B1	
	$\therefore$ equation of normal at $x=1$ $\frac{y+6}{x-1}=\frac{1}{8}$	M1	
	$y+6=\frac{1}{8}x-\frac{1}{8}$		
	$y=\frac{1}{8}x-6\frac{1}{8}$	A1	
		10	

4.3.2 Mathematics Alternative A Paper 2 (121/2)

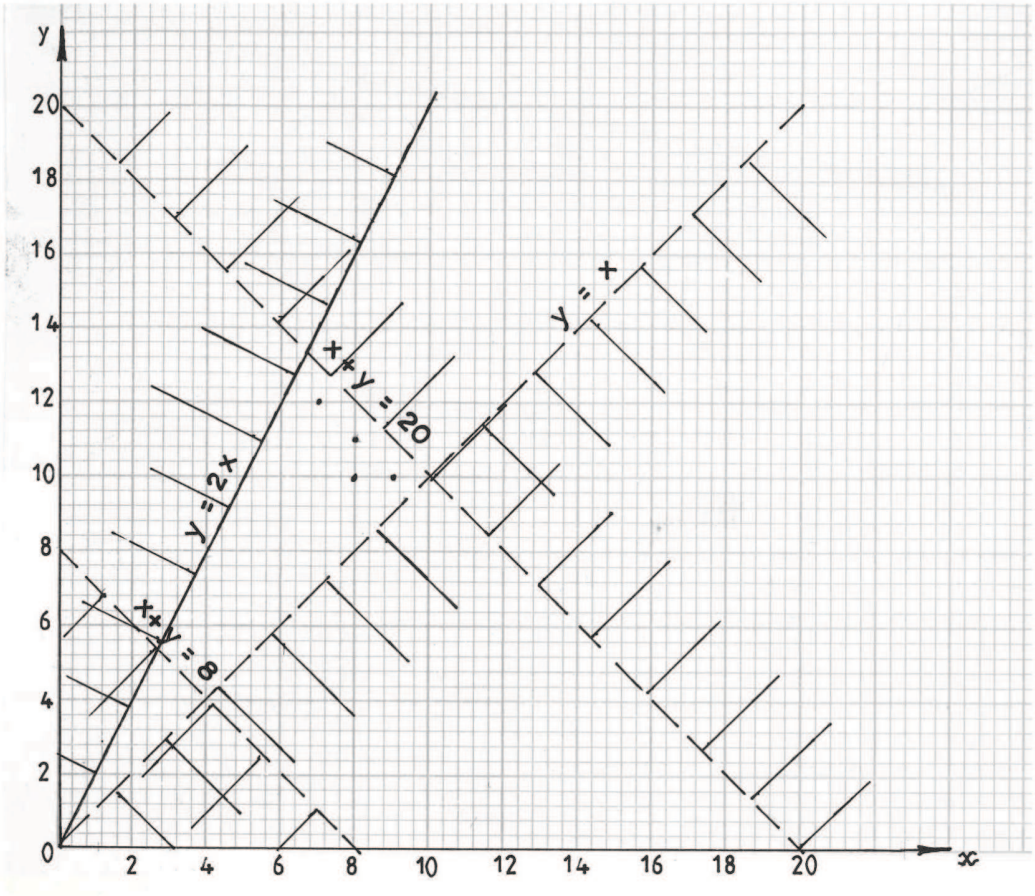
1.	Limits: 12.5 ± 0.05 m and 9.23 ± 0.005 m Maximum difference $= 12.55 - 9.225$ $= 3.325$ m	B1 M1 A1 3	
2.	a) First 6 terms $-7, -4, -1, 2, 5, 8$ b) Sum of 1st 50 terms $S_{50} = \frac{50}{2}\{2 \times -7 + 49 \times 3\}$ $= 3325$	 B1 M1 A1 3	
3.	a) $\angle BAC = 70^\circ - 30^\circ = 40^\circ$ Reflex $\angle BOC = 360^\circ - 80^\circ$ $= 280^\circ$ b) $\angle ACO = 40^\circ - 30^\circ = 10^\circ$	B1 B1 B1 3	
4.	$L = \frac{kM}{N^2}$ $2 = \frac{k \times 12}{36}$ $k = 6$ $\therefore$ equation $L = \frac{6M}{N^2}$	B1 M1 A1 3	
5.	<table><tr><th>Marks</th><th>Frequency</th><th>c.f</th></tr><tr><td>1 - 10</td><td>2</td><td>2</td></tr><tr><td>11 - 20</td><td>4</td><td>6</td></tr><tr><td>21 - 30</td><td>11</td><td>17</td></tr><tr><td>31 - 40</td><td>5</td><td>22</td></tr><tr><td>41 - 50</td><td>3</td><td>25</td></tr></table> Median $= 20.5 + \frac{12.5 - 6}{11} \times 10$ $= 20.5 + 5.91$ $= 26.41$ $\simeq 26$	 B1 M1 M1 A1 4	for c.f

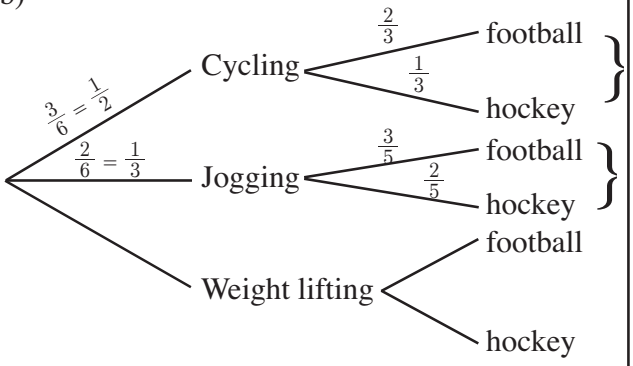
6.	Amplitude = 2 Period $= \frac{360}{3} = 120°$	B1 B1 2	
7.	Area scale factor $= \frac{30}{5} = 6$ $4x - 2x + 2 = 6$ $2x = 4$ $x = 2$	B1 M1 A1 3	
8.	$(3-x)^7 = 3^7 - 7(3)^6x + 21(3)^5x^2 - 35(3)^4x^3 + 35(3)^3x^4 + ...$ $= 2187 - 5103x + 5103x^2 - 2835x^3 + 945x^4$ $(2.8)^7 = (3 - 0.2)^7$ $= 2187 - 5103(0.2) + 5103(0.2)^2 - 2835(0.2)^3 + 945(0.2)^4$ $= 1349.352$	B1 M1 A1 3	
9.	$\text{Log}\frac{15^2}{x} = \log 5(x-4)$ $\frac{15^2}{x} = 5(x-4)$ $x^2 - 4x - 45 = 0$ $(x - 9)(x + 5) = 0$ $x = 9$ or -5 $x = 9$	M1 M1 M1 A1 4	
10.	$PR = \sqrt{60^2 + 11^2} = 61$ $\text{Tan}\,\theta = \frac{10}{61}$ $\theta = 9.31°$	B1 M1 A1 3	

11.	$3x - y = 9$ ..... $\times x$ $x^2 - xy = 4$ $3x^2 - xy = 9x$ $x^2 - xy = 4$ $2x^2 = 9x - 4$ $2x^2 - 9x + 4 = 0$ $(2x - 1)(x - 4) = 0$ $x = \frac{1}{2}$ or $x = 4$ $y = 3\left(\frac{1}{2}\right) - 9$ or $3(4) - 9$ $= -7\frac{1}{2}$ or 3	M1 M1 A1 B1 4	Attempt to solve Factors
12.	$\left(1 + \frac{r}{100}\right)^4 = \frac{495000}{280000}$ $1 + \frac{r}{100} = 1.153$ $r = 15.3$	M1 M1 A1 3	
13.	$8008 = \frac{40 + \theta}{360} \times 2 \times \frac{22}{7} \times 6370$ $40 + \theta = \frac{8008 \times 360 \times 7}{2 \times 22 \times 6370} = 72$ $\theta = 72^\circ - 40^\circ$ $= 32^\circ$ Position of B(32^o S, 20^oW)	M1 M1 A1 3	or 32^o seen
14.	$\underset{\sim}{r} + \underset{\sim}{s} = (7\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}) + (-\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k})$ $= 6\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ $\lvert \underset{\sim}{r} + \underset{\sim}{s} \rvert = \sqrt{6^2 + 3^2 + (-2)^2}$ $= 7$	B1 M1 A1 3	

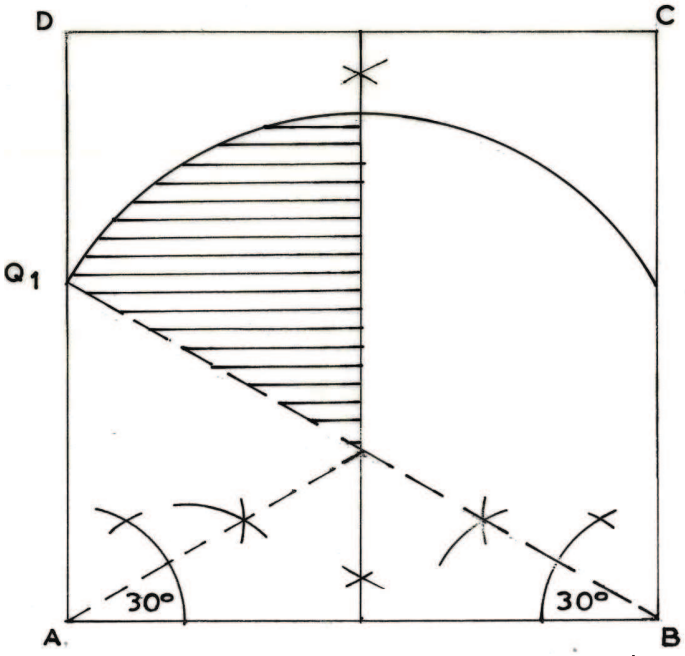
15.	$y = \int (x^2 - 4x + 3)\, dx$ $= \frac{1}{3}x^3 - 2x^2 + 3x + c$ $0 = \frac{1}{3} - 2 + 3 + c$ $\therefore c = -\frac{4}{3}$ $\therefore y = \frac{1}{3}x^3 - 2x^2 + 3x - \frac{4}{3}$	M1 M1 A1 3	
16.	Temperature at the 2nd minute = 60^o Temperature at the 11th minute = 18^o Average rate of cooling $= \frac{60 - 18}{2 - 11}$ $= \frac{42}{\lvert 9 \rvert}$ $= 4\frac{2}{3}$ C/ min	B1 M1 A1 3	for both √
17.	a) $A = \frac{3}{4}B, C = 2B$ $\Rightarrow A:B:C = \frac{3}{4}B:B:2B$ $= 3:4:8$ b) $\left(\frac{168}{8} \times 4\right)$litres $= 84\ l$ c) (i) $\frac{3 \times 160 + 4 \times 205 + 8 \times 100}{3 + 4 + 8}$ $=$ Ksh 140 (ii) $\frac{182 - 140}{140} \times 100\%$ $= 30\%$ (iii) Ksh $140 \times \frac{125}{100}$ $=$ Ksh 175	M1 A1 M1 A1 M1 A1 M1 A1 M1 A1 10	

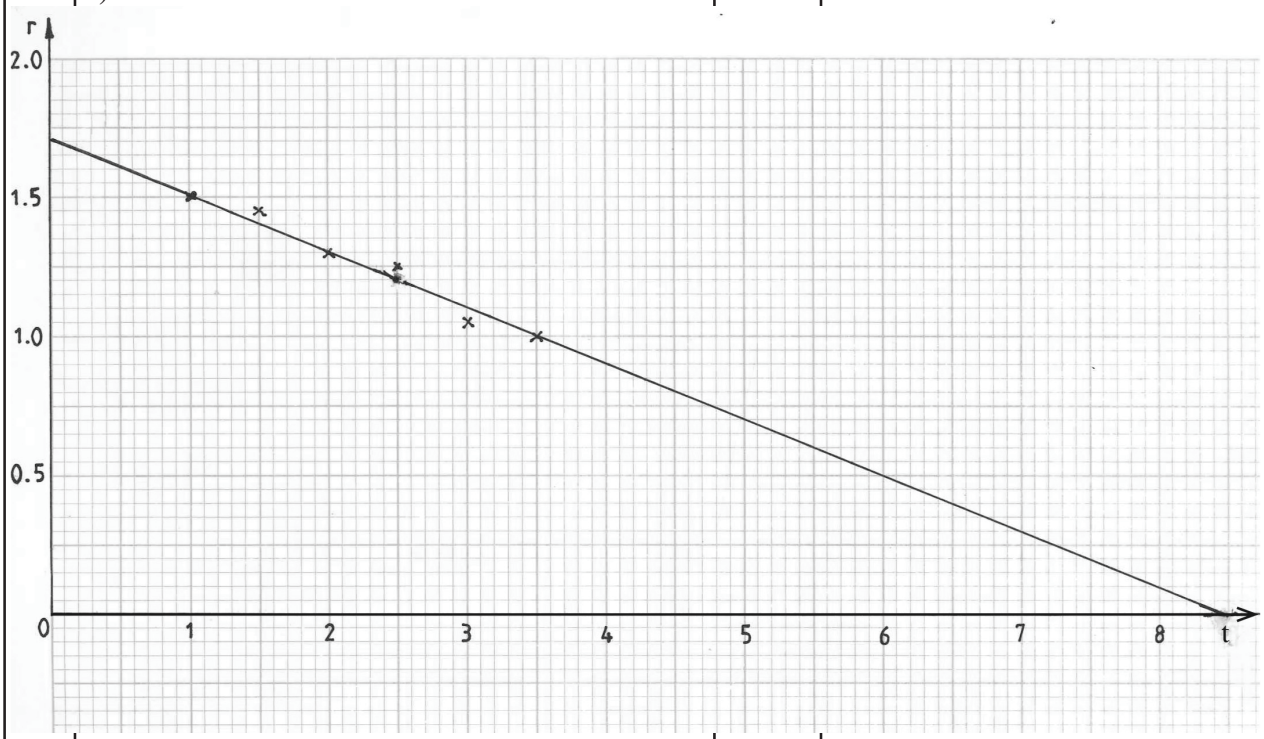
18.	a) (i) $(50 + 40)(50) = 30(30 + x)$ $4500 = 900 + 30x$ $30x = 3600$ $QS = x = 120$ cm	M1 A1	
	(ii) $RS = \frac{1}{2}QS$ $= \frac{1}{2}(120) = 60$ cm $OR = \sqrt{61^2 - 60^2}$ $= 11$ cm	B1 M1 A1	
	b) (i) $\text{Sin}\,\theta = \frac{60}{61}$ $\theta = 79.6°$	M1 A1	or equivalent
	(ii) Angle at the centre $= 2 \times 79.6$ $= 159.2^o$	M1	
	Length of minor arc QS $= \frac{159.2}{360} \times 2\pi \times 61$ $= 169.5$ cm	M1 A1	
		10	
19.	a) (i) $38392 + 2108$ $=$ Ksh 41000	M1 A1	
	(ii) $10164 \times 0.1 + 9576 \times 0.15 + 9576 \times 0.2$ $+ 9576 \times 0.25 + 2108 \times 0.3$ $= 1016.4 + 1436.4 + 1915.2 + 2394 + 632.4$ $=$ Ksh 7394.4	M1 M1 M1 A1	$\surd$ 1st band $\surd$ 3 middle bands $\surd$ last (5th) band
	monthly income tax $= 7394.4 - 1162$ $=$ Ksh 6232.4	B1	
	b) Amount saved in coop society $= \frac{5}{100} \times (41000 - 15000)$ $=$ Ksh1300	M1	
	Nett pay 41000 - (6232.4 + 1300) = Ksh 33467.6	M1 A1	
		10	

20.	a) $y > x$ $y \leq 2x$ $x + y < 20$ $x + y > 8$ b) (i)	B1 B1 B1 B1	
	 		
		B1 B1 B1 B1	line y = 2x and √ shading broken line x + y = 20 and √ shading broken line x + y = 8 and √ shading broken line y = x and √ shading
	(ii) Maximum area: 9×10 $= 90\ m^2$	M1 A1	
		10	

21.	a) (i) $\frac{3}{6}+\frac{1}{6}$	M1	
	$=\frac{2}{3}$	A1	
	(ii) $\frac{2}{6}\times\frac{2}{6}$	M1	
	$=\frac{1}{9}$	A1	
	b) 	B1 B1	
	c) (i) P(Gataro plays football)		
	$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{3}\times\frac{3}{5}+\frac{1}{6}\times\frac{1}{2}$	M1	
	$=\frac{37}{60}$	A1	
	(ii) P(neither jogs nor plays football)		
	$=\frac{1}{2}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{2}$	M1	
	$=\frac{1}{4}$	A1	
		10	

22.	a) (i) $\underset{\sim}{BA} = \underset{\sim}{a} - \underset{\sim}{b}$	B1	
	(ii) $\underset{\sim}{BN} = \frac{1}{3}\underset{\sim}{BA} = \frac{1}{3}(\underset{\sim}{a} - \underset{\sim}{b})$	B1	
	(iii) $\underset{\sim}{ON} = \underset{\sim}{b} + \frac{1}{3}(\underset{\sim}{a} - \underset{\sim}{b})$	M1	
	$= \frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$	A1	
	b) $\underset{\sim}{BX} = h\underset{\sim}{BM} = h\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right)$		
	$\underset{\sim}{OX} = k\underset{\sim}{ON} = k\left(\frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right)$	B1	
	also		
	$\underset{\sim}{OX} = \underset{\sim}{OB} + \underset{\sim}{BX}$		
	$= \underset{\sim}{b} + h\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right)$	B1	
	$k\left(\frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right) = \underset{\sim}{b} + h\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right)$	M1	
	$\frac{1}{3}k\underset{\sim}{a} = \frac{1}{2}h\underset{\sim}{a}$		
	$\frac{1}{3}k = \frac{1}{2}h \Longrightarrow k = \frac{3}{2}h$.....(i)	M1	
	$\frac{2}{3}k\underset{\sim}{b} = \underset{\sim}{b} - h\underset{\sim}{b}$		
	$\frac{2}{3}k = 1 - h$.....(ii)		
	Substituting $k = \frac{3}{2}h$ in (ii)		
	$\frac{2}{3}\left(\frac{3}{2}h\right) = 1 - h \Longrightarrow h = \frac{1}{2}$	M1	
	Substituting $h = \frac{1}{2}$ in (i)		
	$k = \frac{3}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$	A1	for $h = \frac{1}{2}$ and $k = \frac{3}{4}$
		10	

23.			
	(i)	B2	locus of P
	(ii)	B1	construction of 30°
		B1	identification of centre
		B1	drawing of arc
	b) (i) $9.2 \times 10 = 92$ m	B1	
	(ii) area of region bounded by locus of P, locus of Q and line BQ_1	B1	Identifying region
	angle = 60° radius = 46 m	B1	for radius and angle of sector
	$= \pi \times 46^2 \times \frac{60}{360}$	M1	
	$= 1107.94$		
	$\simeq 1108\ m^2$	A1	
		10	

24.	a) 		
		S1 P2 L1	√ scale (P1 for 4 points √ plotted) √ line
	b) (i) value of a $= \frac{-0.7}{3.5}$	M1	
	$= -0.2$	A1	
	value of k = 1.7	B1	
	(ii) equation: $r = -0.2t + 1.7$	B1	
	(iii) value of t when r = 0 $\therefore 0 = -0.2t + 1.7$ $0.2\,t = 1.7$	M1	
	$t = \frac{1.7}{0.2} = 8.5$	A1	
		10	