KENYA NATIONAL EXAMINATION COUNCIL

KCSE 2009

MATHEMATICS

PAPER 1

MARKING SCHEME

Q4. Distance covered by bus

$= 63 \times (10.45 - 8.15)$

$= 63 \times 2.5$

$= 157.5$

Speed of car $\frac{157.5}{1.75}$

$= 90$ km/h

Alt. 1.

Extra distance

$63 \times \frac{3}{4} = 47.25$ M_1

Relative speed

$x - 63$ M_1

$\frac{47.25}{x-63} = \frac{7}{4}$ M_1

A_1

$x = 90$

$\frac{63 \times 150}{105} = 90$ km/h

03

5 $64^{-\frac{1}{2}} \times 27000^{\frac{2}{3}}$

$2^{-4} \times 3^0 \times 5^2$

$= \frac{1}{\sqrt{64}} \times (\sqrt[3]{27000})^2$ M_1 Removal of -ve indices

$\frac{1}{16} \times 3^0 \times 25$ M_1 Removal of roots.

$= \frac{1}{8} \times 900 \times \frac{16}{25}$ A_1 Simplification

$= 72$ B_1

04

6 $AC = \sqrt{85^2 - 75^2} = \sqrt{1600}$ M_1

$= 40$

Area of quad. ABCD

$\frac{1}{2} \times 40 \times 75 + \sqrt{75(75-60)(75-50)(75-40}$ M_1 Sum of both areas

$= 1500 + \sqrt{984375}$

$= 1500 + 992$

$= 2492\ m^2$ A_1

$\simeq 0.25$ ha B_1

04

7. Time between Monday 0545h and Friday 1945h

$= 4 \times 24 + 14$ M_1 Expression leading to 110hrs

$= 110$ h

Time lost $= 0.5 \times 110 = 55$ Min

Time shown in 12hour system

$1945 - 55 = 1850$ h M_1

$= 6.50$ p.m A_1

03

8 $\frac{12x^2 + 9x - 6a^2}{9x^2 - 4a^2}$

$\frac{(4x + 3a)(3x - 2a)}{(3x + 2a)(3x - 2a)}$

$= \frac{4x + 3a}{3x + 2a}$

M1 √ Factorising numerator fully

M1 " denominator √ly.

A1

03

9 $y = -\frac{2}{5}x + 2$

gradient $= -\frac{2}{5}$

$\frac{k - 5}{3 - -2} = -\frac{2}{5}$

$k - 5 = -2$

$\Rightarrow k = 3$

$5y + 2x = 10$

B1 $5(5) + 2(-2) = 21$

M1 $5k + 6 = 21$

(k,3) $5k = 15$

$k = 3$

A1

03

10 let exterior ∠ = ∠at the centre) be x

$\therefore 6.5x + x = 180$

$7.5x = 180$

$x = 24°$

$\therefore$ No. of sides $= \frac{360}{24}$

$= 15$ sides

Alt. 1.

M1 let the no. of sides = n

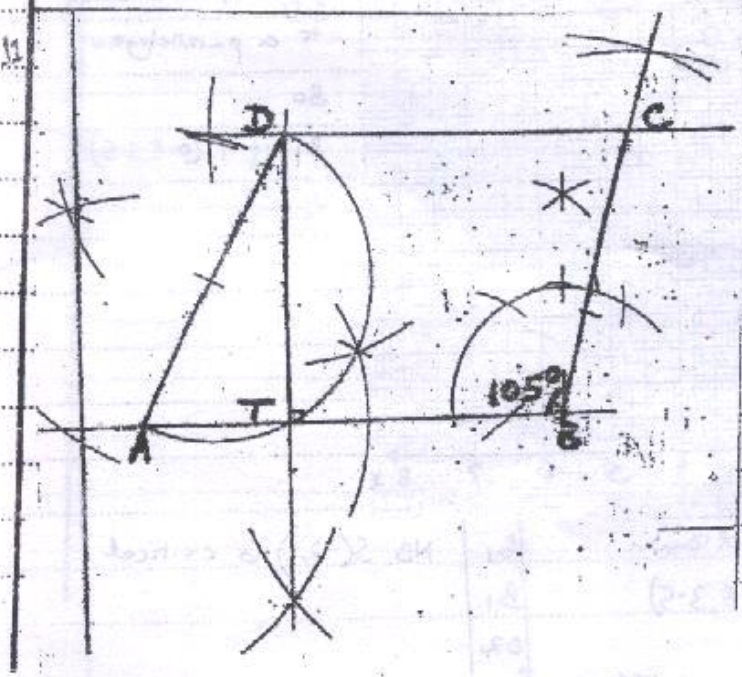
$\frac{360}{n} \times 6.5 + \frac{360}{n} = 180$ M1 M1

M1 $180n = 2700$

A1 $n = 15$ A1

03

11



- construction of 105° B1
- fixing point C and constructing line parallel to AB through C B1
- completion of trapezium ABCD B1
- location of point T from D only if any above B above list B1

04

12. let angle between ground and wire be $\theta°$

$\therefore \theta + \frac{1}{3}\theta = 90$

$\Rightarrow \theta = 90 \times \frac{3}{4} = 67.5$ B1 or 22.5°

let the wire be x cm in length

$\therefore \cos 67.5 = 6/x$

$x = \frac{6}{\cos 67.5} = \frac{6}{0.38268432}$ M1

$= 15.68$m or 1568cm A1

03

13. $\sin(3x+30) = \sin 60°$ B1 for 60°

$\sin(3x+30) = \sin 120°$ B1 for 120°

$3x+30 = 60$

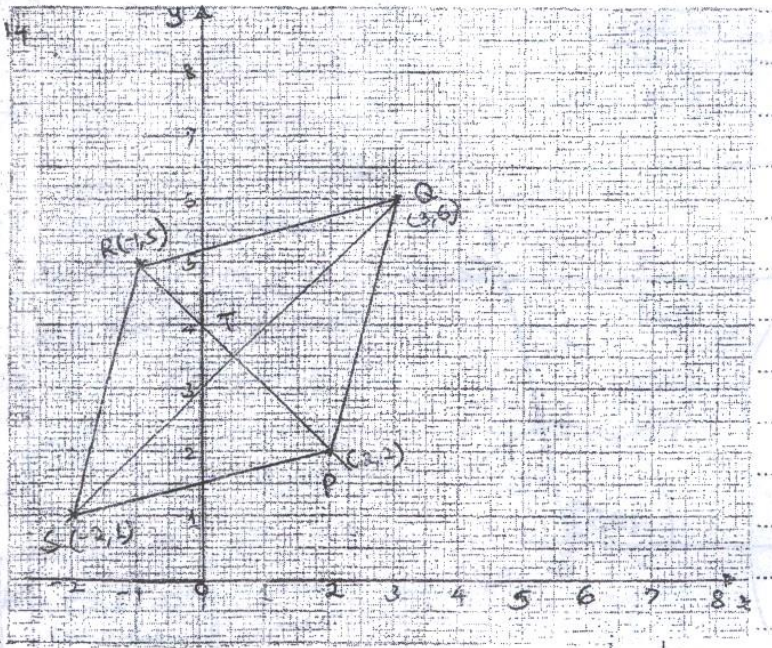
$3x+30 = 120$

$x = 10°$, $x = 30°$ B1 for 10°

B1 for 30°

04

14.



Different scales on x and y axis leads to a parallelogram.

B0

B1 if T(0.5, 3.5)

(a) Rhombus PQRS Vly drawn B1 NB S(-2,1) is critical

(b) Co-ordinates of T(0.5, 3.5) B1

02

15 Commission earned

$0.225 \times 0.2 \times 3800$

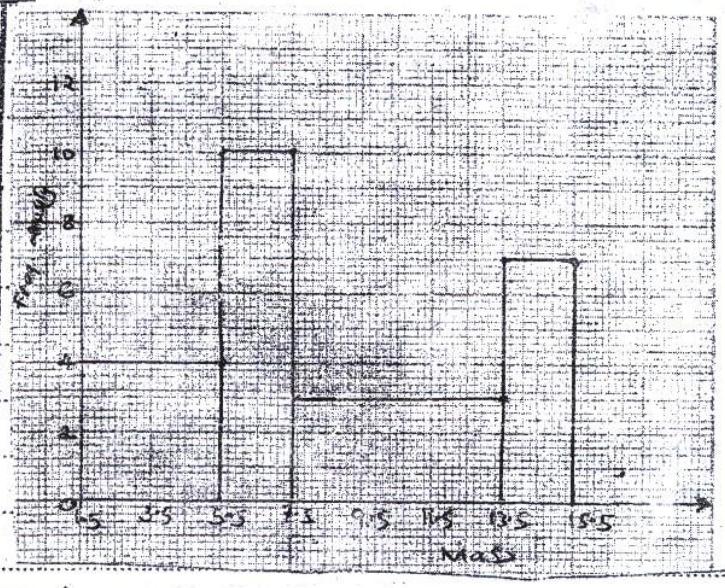
171

M1

A1

02

16



Frequency density = $\frac{\text{freq.}}{\text{C. Width}}$

1.5 – 5.5 ✓ bar B1

5.5 – 7.5 ✓ bar B1

7.5 – 13.5 bar B1

Check weights

Check weights

1.5 – 5.5 — 4 ✓

5.5 – 7.5 — 10 ✓

7.5 — 13.5 — 3 ✓

13.5 – 15.5 — 7 given.

17. a) $BC^2 = 6^2 + 8^2 - 2\times6\times8\times\cos 5°$ — M1 — If drawn to scale the line DB is not a straight line there are so many possibilities

$= 100 - 61.71$

$BC = \sqrt{38.2912} = 6.19$ — A1

b) let $\angle ABC$ be $\beta°$

$\frac{\sin\beta}{6} = \frac{\sin 50°}{6.19}$; $\sin\beta = \frac{6\sin 50}{6.19}$ — M1 — Accept 48.16, 48.15

$\therefore \beta = 47.95°$ — M1 — 47.94°, 47.96° depending on Method used. — A1

c) let $\angle CAD$ be $\alpha°$

$2.82^2 = 7^2 + 6^2 - 2\times7\times6\cos\alpha$ — M1

$\cos\alpha = \frac{49+36-7.9524}{84}$ — M1

$\alpha = 23.48°$. — A1 — 22.84 is possible

d) Area of $\triangle ACD = \frac{1}{2}\times7\times6\times\sin 23.48°$ — M1 — Heron's formula $S = 7.91$

$\sqrt{7.91\times1.42\times2.91\times5.10}$

$= 8.37$

$= 8.37\ cm^2$ — A1

10

18. (a) (i) Modal Class 60–69 — B1

(ii) Class where Median mark lies

0–9	
10–19	3
20–29	7
30–39	14
40–49	24
50–59	40 x
60–69	60
70–79	66
80–89	69
90–99	70

Median is 35

Median class is 50–59 — B1

class	centres x	fd	d = x − 1
0–9	4.5	−49.9	−46.9
10–19	14.5	−79.8	−39.9
20–29	24.5	−119.6	−29.9
30–39	34.5	−139.3	−19.9
40–49	44.5	−99.0	−9.9
50–59	54.5	1.6	0.1
60–69	64.5	20.2	10.1
70–79	74.5	120.6	20.1
80–89	84.5	90.3	30.1
90–100	94.5	40.1	40.1

B1 ✓ for Cf column or implied

B1 ✓ class centres

M1 ✓ deviations

B1 ✓ fd.

$\Sigma f = 70$ — B1

$\Sigma fd = -33$ — A1

Mean $= 54.4 - \frac{33}{70}$ — M1

$= 53.93$ — A1

9(c)(i) Original price $= \frac{16200}{x}$	B1	
(ii) Price after discount $\frac{16200}{x+3}$	B1	
b)(i) $\frac{16200}{x} - 60 = \frac{16200}{x+3}$	M1	or equivalent
$\Rightarrow (16200 - 60x)(x+3) = 16200x$	M1	Removal of denominators
$16200x + 48600 - 60x^2 - 180x = 16200x$		
$60x^2 + 180x - 48600 = 0$	M1	For quadratic equation
$x^2 + 3x - 810 = 0$		
$(x+30)(x-27) = 0$	M1	For factorization of quad. eqn.
$x = -30$ or $x = 27$		
No. of calculators bought = 30	A1	C.A.O or avoids -ve
(c) Initial cost of calculators		
$\frac{16200}{27} = 600$	M1	
Discount offered as a percentage		
$\frac{\frac{16200}{27} - \frac{16200}{30}}{600} \times 100$	M1	
$= 10\%$	A1	
	10	
20 a)(i) $\underset{\sim}{ON} = \frac{1}{2}\begin{pmatrix}-8\\5\end{pmatrix} = \begin{pmatrix}-4\\2.5\end{pmatrix}$		Alt. 20 (a)(ii) $\vec{MN} = \begin{pmatrix}-6\\2.5\end{pmatrix}$ B1
N is $(-4, 2.5)$	B1	$\lvert\vec{MN}\rvert = \sqrt{-6^2 + 2.5^2}$ M1
$M = \left(\frac{-8+12}{2}, \frac{5-5}{2}\right)$	M1	$= 6.5$ A1
M is $(2, 0)$	A1	
(ii) $\underset{\sim}{NM} = \begin{pmatrix}6\\-2\frac{1}{2}\end{pmatrix}$	B1	
$\lvert NM\rvert = \sqrt{6^2 + (2.5)^2}$	M1	
$= 6.5$	A1	
(b) $\underset{\sim}{OB} = \begin{pmatrix}12\\-5\end{pmatrix}$, $\underset{\sim}{NM} = \begin{pmatrix}6\\-2\frac{1}{2}\end{pmatrix}$		Apply OW^{-1} If vector SQn missing
$\therefore NM = \frac{1}{2}OB$	B1	
(c) $\underset{\sim}{OP} = \begin{pmatrix}2\\0\end{pmatrix} + 2\begin{pmatrix}-6\\2\frac{1}{2}\end{pmatrix} = \begin{pmatrix}-10\\5\end{pmatrix}$	M1	
$OP' = \begin{pmatrix}-10\\5\end{pmatrix} + \begin{pmatrix}-5\\8\end{pmatrix} = \begin{pmatrix}-15\\13\end{pmatrix}$	M1	
$\therefore P'$ is $(-15, 13)$	A1	
	10	

21 (a) Volume of water $\frac{6}{9+x} = \frac{2}{x}$ $x = 4.5$

$\therefore$ Vol $= \frac{1}{3} \times 3.142 (6^2 \times 13.5 - 2^2 \times 4.5)$

$= 508.94 - 18.25 = 490.09$

(b)(i) Volume of Sphere

Top radius $\frac{r}{14.5} = \frac{2}{4.5}$ $r = 6.44$

Volume $= \frac{1}{3} \times 3.142 (6.444^2 \times 14.5 - 6^2 \times 13.5)$

$= 121.6 \Rightarrow 121\frac{11}{15}$

(ii) $\frac{4}{3} \pi x^3 = 121.6$

$x^3 = 121.6 \times \frac{3}{4\pi}$

$x = 3.073$

Follow through

ALT 1:

LSF $14.5 : 4.5$

$29 : 9$

VSF $24389 : 729$

$\frac{24389}{729} = \frac{V}{18.849}$ (M1)

(A1)

$V = \frac{24389}{729} \times 18.849 = 630.619$ (M1)

Volume of marble

$= 630.619 - 508.938$ (M1)

$= 121.681$ (A1)

b(i) $V = \frac{1}{3} h (R^2 + Rr + r^2)$

$= \frac{9}{3} (6^2 + 6 \times 2 + 2^2)$ (M1M1)

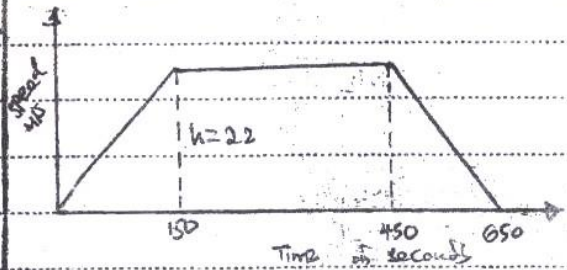
$= 156 \pi = 490.088$ (A1)

(ii) $\frac{1}{3} \times \frac{22}{7} (6.44^2 \times 14.5 - 6^2 \times 13.5)$

$= 630.79 - 509.14$

$= 121.65$

22



(a) $\frac{1}{2} \times 150h + \frac{1}{2} \times 200h + 300h = 10450$ M1 or equivalent.

$475h = 10450$

$h = 22$ m/s A1

Max speed $= \frac{22 \times 60 \times 60}{1000} = 79.2$ km/h B1 C.A.O

(b) Acceleration $\frac{22}{150}$ m/s M1

$= \frac{11}{75}$ A1 or 0.1467 m/s^2 or 1900.8 km/h^2

(c) $\frac{1}{2} \times 100 \times 11$ M1

$= 550$ A1

(d) Time for half of journey

$\frac{1}{2} \times 22 (150 + t + t) = \frac{1}{2} \times 10450$ M1

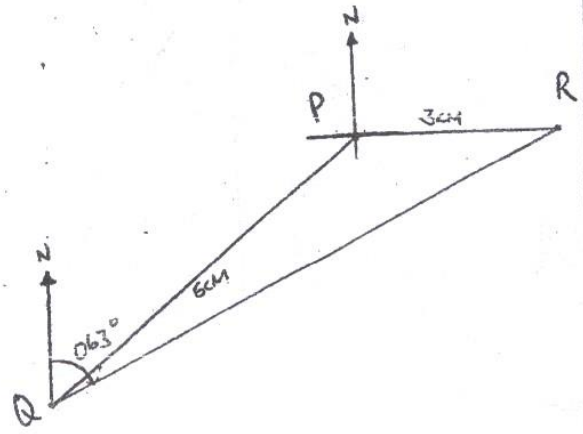
$t = 162.5$ A1

Total time $= 150 + 162.5$

$= 312.5$ B1

10

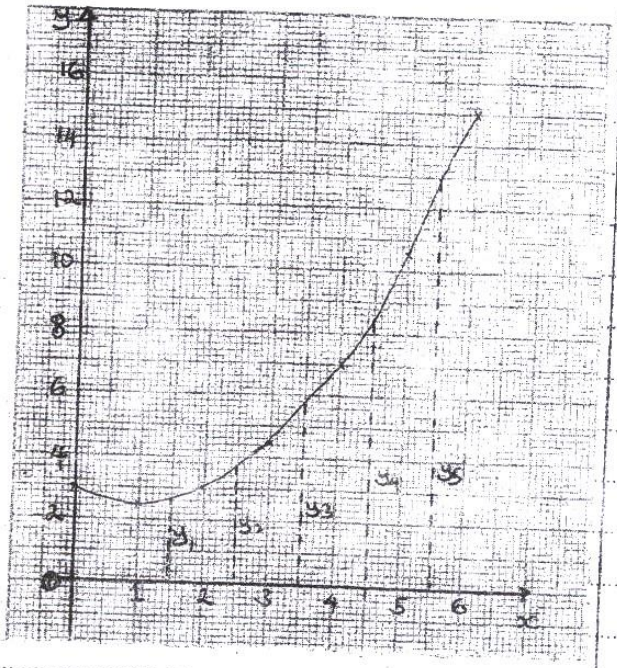
23



a) Direction and distance of Q from P	B_1	
- Direction and distance of R from P	B_1	
(i) Distance conversion		
8.5×40	M_1	8.5 ± 0.1 . $8.4 \times 40 = 336$ or $8.6 \times 40 = 344$
$= 340$	A_1	
(ii) ✓ North line at Q	B_1	
bearing 063° stated	B_1	$\pm 1°$ N 62°E or N63°E or N64°E
(i) Distance from the top of the post at Q to the tip of post at P		Follow through
$x = \frac{240}{\cos 9°}$ or $x \cos 9° = 240$	M_1	if calculation
$= 243M$	A_1	B_1 for calculated angle
		B_1 for angle in form of bearing.
(ii) Speed of bird		
$\frac{243 \times 60 \times 60}{1000 \times 18}$	M_1	$\frac{240}{\sin 81°} = x$
$= 48.6$ km/h	A_1	
	10	

x	0	1	2	3	4	5	6
$y=\frac{1}{2}x^2-x+3$	3	$2\frac{1}{2}$	3	$4\frac{1}{2}$	7	$10\frac{1}{2}$	15

B_1



P1 ✓✓ double tick for at least 5 correct scores on table

C1 C.A.O

(b) When $y_1 \ldots y_5$ are read from the graph

B0

M0

A0

$y_1 = \frac{1}{2}\times 1.5^2 - 1.5 + 3 = 2.625$

$y_2 = \frac{1}{2}\times 2.5^2 - 2.5 + 3 = 3.625$

$y_3 = \frac{1}{2}\times 3.5^2 - 3.5 + 3 = 5.625$

$y_4 = \frac{1}{2}\times 4.5^2 - 4.5 + 3 = 8.625$

$y_5 = \frac{1}{2}\times 5.5^2 - 5.5 + 3 = 12.625$

B1 ✓ ordinates calculated

Approximate area $= 1(2.625+3.625+5.625+8.625+12.625)$ M1

$= 33.125$ Square units A1

Area $= \int_1^6 (\frac{1}{2}x^2 - x + 3)\,dx = \left[\frac{x^3}{6} - \frac{x^2}{2} + 3x\right]_1^6$ M1

$= \left[\frac{6^3}{3} - \frac{6^2}{2} + 3\times 6\right] - \left[\frac{1^3}{6} - \frac{1^2}{2} + 3\right] = 33.3$ M1

% error $\frac{33.3 - 33.125}{33.3} \times 100 = 0.615\%$ M1 A1

$\frac{33\frac{1}{3} - 33\frac{1}{8}}{33\frac{1}{3}} \times 100\% = 0.625\%$

151 10